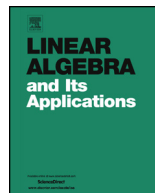




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On the numerical range of some weighted shift matrices and operators


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ABSTRACT

In this paper, we compute and compare the numerical radii of certain weighted shift matrices. Also we compute the numerical radius of a weighted shift operator on the Hardy space H^2 . The purpose of this paper is to develop results in [7].

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1. Introduction

Let A be a bounded linear operator on a complex Hilbert space H . The *numerical range* of A is defined as the set

$$W(A) = \{ \langle Ax, x \rangle : \|x\| = 1, x \in H \},$$

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where $\langle \cdot, \cdot \rangle$ and $\| \cdot \|$ denote the standard inner product and its associated norm in H . It is known that $W(A)$ is a nonempty, bounded and convex subset of \mathbb{C} , see for example [12]. The numerical radius $w(A)$ of an operator A is given by

$$w(A) = \sup\{|\lambda|, \lambda \in W(A)\}.$$

For its other properties, see [12].

We consider a weighted shift operator A on the Hilbert space $\ell^2(\mathbb{N})$ defined by

$$A = A(a_1, a_2, \dots) = \begin{pmatrix} 0 & 0 & 0 & 0 & \dots \\ a_1 & 0 & 0 & 0 & \dots \\ 0 & a_2 & 0 & 0 & \dots \\ 0 & 0 & a_3 & 0 & \dots \\ \vdots & \vdots & \vdots & \ddots & \ddots \end{pmatrix} \tag{1}$$

where $\{a_n\}$ is a bounded sequence. Define a unitary operator

$$U = \text{diag}(s_1, s_1 s_2, s_1 s_2 s_3, \dots),$$

with $s_1, s_{n+1} = \overline{a_n}/a_n$ if $a_n \neq 0$, and $s_{n+1} = 1$ if $a_n = 0$. Then

$$UAU^* = |A|,$$

where $|A|$ is the entrywise absolute value $(|a_{ij}|)$ of the matrix $A = (a_{ij})$.

Hence, we can always assume the weights of a weighted shift operator are nonnegative. It is known that the numerical range of a weighted shift operator is a circular disk about the origin, see for example [15] and [16] and the numerical range of a weighted shift matrix is a closed circular disk centered at the origin, see for example [8].

In particular, $W(A(1, 1, \dots))$ is an open unit circular disk (see [16]). Further, Berger and Stampfli [1] showed that if $(1 + h) > \sqrt{2}$, then

$$w(A(1 + h, 1, 1, \dots)) = \frac{1}{2}((1 + h)^2 - 1)^{1/2} + ((1 + h)^2 - 1)^{-1/2}.$$

Recently, Chien and Sheu [7] showed that if $(1 + h) > \frac{\sqrt{6}}{2}$, then

$$w(A(1, 1 + h, 1, \dots)) = \frac{1}{2} \left(((h(2 + h) + \sqrt{(h(2 + h))^2 + 4h(2 + h)})/2)^{1/2} + ((h(2 + h) + \sqrt{(h(2 + h))^2 + 4h(2 + h)})/2)^{-1/2} \right),$$

also if $(1 + h) > \sqrt{2}$ then they showed that

$$w(A_1) < w(A_2),$$

where $A_1 = A(1 + h, 1, 1, \dots)$ and $A_2 = A(1, 1 + h, 1, 1, \dots)$.

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