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On the numerical range of some weighted shift matrices and operators

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ABSTRACT

In this paper, we compute and compare the numerical radii of certain weighted shift matrices. Also we compute the numerical radius of a weighted shift operator on the Hardy space H^2. The purpose of this paper is to develop results in [7].

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1. Introduction

Let A be a bounded linear operator on a complex Hilbert space H. The *numerical range* of A is defined as the set

$$W(A) = \{\langle Ax, x\rangle: \|x\| = 1, \ x \in H\},$$

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where $\langle\cdot,\cdot\rangle$ and $\|\cdot\|$ denote the standard inner product and its associated norm in H. It is known that $W(A)$ is a nonempty, bounded and convex subset of $\mathbb{C}$, see for example [12]. The *numerical radius* $w(A)$ of an operator A is given by

$$w(A) = \sup\{|\lambda|,\ \lambda \in W(A)\}.$$

For its other properties, see [12].

We consider a weighted shift operator A on the Hilbert space $\ell^2(\mathbf{N})$ defined by

$$A = A(a_1, a_2, \ldots) = \begin{pmatrix} 0 & 0 & 0 & 0 & \ldots \\ a_1 & 0 & 0 & 0 & \ldots \\ 0 & a_2 & 0 & 0 & \ldots \\ 0 & 0 & a_3 & 0 & \ldots \\ \vdots & \vdots & \vdots & \ddots & \ddots \end{pmatrix} \tag{1}$$

where $\{a_n\}$ is a bounded sequence. Define a unitary operator

$$U = diag(s_1, s_1 s_2, s_1 s_2 s_3, \ldots),$$

with $s_1, s_{n+1} = \overline{a_n}/a_n$ if $a_n \neq 0$, and $s_{n+1} = 1$ if $a_n = 0$. Then

$$UAU^* = |A|,$$

where $|A|$ is the entrywise absolute value $(|a_{ij}|)$ of the matrix $A = (a_{ij})$.

Hence, we can always assume the weights of a weighted shift operator are nonnegative. It is known that the numerical range of a weighted shift operator is a circular disk about the origin, see for example [15] and [16] and the numerical range of a weighted shift matrix is a closed circular disk centered at the origin, see for example [8].

In particular, $W(A(1,1,\ldots))$ is an open unit circular disk (see [16]). Further, Berger and Stampfli [1] showed that if $(1+h) > \sqrt{2}$, then

$$w\big(A(1+h,1,1,\ldots)\big) = \frac{1}{2}\big(\big((1+h)^2-1\big)^{1/2} + \big((1+h)^2-1\big)^{-1/2}\big).$$

Recently, Chien and Sheu [7] showed that if $(1+h) > \frac{\sqrt{6}}{2}$, then

$$\begin{aligned} w\big(A(1,1+h,1,\ldots)\big) = \frac{1}{2}\Big(&\Big(\Big(h(2+h) + \sqrt{\big(h(2+h)\big)^2 + 4h(2+h)}\Big)/2\Big)^{1/2} \\ &+ \Big(\Big(h(2+h) + \sqrt{\big(h(2+h)\big)^2 + 4h(2+h)}\Big)/2\Big)^{-1/2}\Big), \end{aligned}$$

also if $(1+h) > \sqrt{2}$ then they showed that

$$w(A_1) < w(A_2),$$

where $A_1 = A(1+h,1,1,\ldots)$ and $A_2 = A(1,1+h,1,1,\ldots)$.

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