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Invariant quadrics and orbits for a family of rational systems of difference equations

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ABSTRACT

We study the existence of invariant quadrics for a class of systems of difference equations in $\mathbb{R}^n$ defined by linear fractionals sharing denominator. Such systems can be described in terms of some square matrix A and we prove that there is a correspondence between non-degenerate invariant quadrics and solutions to a certain matrix equation involving A. We show that if A is semisimple and the corresponding system admits non-degenerate quadrics, then every orbit of the dynamical system is contained either in an invariant affine variety or in an invariant quadric.

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1. Introduction

The apparent tractability of rational systems of difference equations in comparison with other non-linear equations and the fact that they frequently appear in applications have increased the interest in the study of such systems and higher order rational difference equations [4,5,8,10]. The determination of geometric invariants of a system of difference equations is, in general, a difficult task and so it is in the case of rational

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equations. In this paper we will find a family of quadratic varieties which remain invariant for a class of systems of difference equations in $\mathbb{R}^n$, $n \geqslant 2$, of the type

$$X(k+1) = \big(F_1\big(X(k)\big), \dots, F_n\big(X(k)\big)\big), \quad X(k) \in \mathbb{R}^n \tag{1}$$

where the maps F_i are linear fractionals sharing denominator:

$$F_i(X) = \frac{a_{i1}x_1 + a_{i2}x_2 + \dots + a_{in}x_n + c_i}{b_1x_1 + b_2x_2 + \dots + b_nx_n + d}, \quad i = 1, 2, \dots, n,$$

where $X = (x_1, x_2, \dots, x_n)$ and all the involved parameters are real. Such kind of rational systems have been treated in [3] where global periodicity properties were studied. Further, AlSharawi and Rhouma [1] studied a biological model given by systems of rational difference equations with a common denominator. The key fact for the study of those systems is that they can be written in certain matricial form by the use of homogeneous coordinates. Explicitly, if one denotes by q the mapping given by $q(a_1, a_2, \dots, a_{n+1}) = (a_1/a_{n+1}, a_2/a_{n+1}, \dots, a_n/a_{n+1})$ for $(a_1, a_2, \dots, a_{n+1}) \in \mathbb{R}^{n+1}$ with $a_{n+1} \neq 0$ and $\ell : \mathbb{R}^n \to \mathbb{R}^{n+1}$ is the transformation to homogeneous coordinates given by $\ell(a_1, \dots, a_n) = (a_1, \dots, a_n, 1)^T$, where M^T stands for the transposed of a matrix M, then the system can be written in the form $X(k+1) = q \circ A \circ \ell(X(k))$ for the square matrix of order $(n+1)$ given by

$$A = \left(\begin{array}{c|c} A_1 & C^T \\ \hline B & d \end{array}\right), \tag{2}$$

where $A_1 = (a_{ij})$, $B = (b_1, \dots, b_n)$ and $C = (c_1, \dots, c_n)$.

The following result of [3] will be frequently used:

Lemma 1. *With the above notation, we have:*

(a) $q(\ell(x)) = x$ *for all* $x \in \mathbb{R}^n$.
(b) $\ell(q(a)) = (1/a_{n+1})a$ *for all* $a = (a_1, \dots, a_{n+1})$ *such that* $a_{n+1} \neq 0$.
(c) *If* $a, b \in \mathbb{R}^{n+1}$ *are such that* $q(a)$ *and* $q(b)$ *exist, then* $q(a) = q(b)$ *if and only if* $a = \lambda b$ *for some* $\lambda \in \mathbb{R}$, $\lambda \neq 0$.
(d) *If* A *is an* $(n+1) \times (n+1)$ *matrix, then* $q(A\ell(q(a))) = q(Aa)$ *whenever both members exist.*

Since we will be interested in orbits and invariant sets, we recall the following:

Let $F : \Omega \subset \mathbb{R}^n \to \mathbb{R}^n$ be a map defined on certain non-empty set Ω and $X(0) \in \mathbb{R}^n$. In we denote by F^k the k-th power under composition of F, then the *orbit* of X_0 is the sequence $\{F^k(X_0)\}_{k \in \mathbb{N}}$ or, equivalently, the solution of the system $X(k+1) = F(X(k))$ with initial condition $X(0) = X_0$. If for some $k \in \mathbb{N}$ the corresponding power $F^k(X_0)$

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