

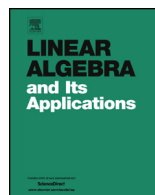


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Graphs associated with matrices over finite fields and their endomorphisms



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ABSTRACT

Let $\mathbb{F}^{m \times n}$ be the set of $m \times n$ matrices over a field \mathbb{F} . Consider a graph $G = (\mathbb{F}^{m \times n}, \sim)$ with $\mathbb{F}^{m \times n}$ as the vertex set such that two vertices $A, B \in \mathbb{F}^{m \times n}$ are adjacent if $\text{rank}(A - B) = 1$. We study graph properties of G when \mathbb{F} is a finite field. In particular, G is a regular connected graph with diameter equal to $\min\{m, n\}$; it is always Hamiltonian. Furthermore, we determine the independence number, chromatic number and clique number of G . These results are used to characterize the graph endomorphisms of G , which extends Hua's fundamental theorem of geometry on $\mathbb{F}^{m \times n}$.

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Independence number
Chromatic number

1. Introduction

Let \mathbb{F} be a field and $\mathbb{F}^{m \times n}$ the set of $m \times n$ matrices over \mathbb{F} . Define a metric d on $\mathbb{F}^{m \times n}$ by

$$d(A, B) = \text{rank}(A - B).$$

Two matrices $A, B \in \mathbb{F}^{m \times n}$ are *adjacent*, denoted by $A \sim B$, if $d(A, B) = \text{rank}(A - B) = 1$. This metric and adjacency relation give rise to an interesting geometrical structure on $\mathbb{F}^{m \times n}$.

In mid 1940s, Hua initiated the study of the fundamental theorem of the geometry of matrices that concerns the characterization of maps $\phi : \mathbb{F}^{m \times n} \rightarrow \mathbb{F}^{m \times n}$ leaving invariant the adjacency relation, i.e., $\text{rank}(\phi(A) - \phi(B)) = 1$ whenever $\text{rank}(A - B) = 1$. Hua also considered the problem on matrices over a division ring, and his study generated considerable interest and led to many interesting results; for example, see [8,9,11,13,18,20].

Suppose \mathbb{F} is the finite field \mathbb{F}_q with q elements. Then the adjacency relation $A \sim B$ in $\mathbb{F}_q^{m \times n}$ defined above, i.e., $A \sim B$ if $\text{rank}(A - B) = 1$, gives rise to a graph $G = (V, \sim)$ with $V = \mathbb{F}_q^{m \times n}$ as the vertex set and there is an edge joining $A, B \in V$ if and only if $A \sim B$. We call $G = (\mathbb{F}_q^{m \times n}, \sim)$ a *matrix graph*, which is also called a *bilinear forms graph* in graph theory. This graph has a lot of interesting properties. For example, it is easy to check that G is a regular graph with diameter equal to $\min\{m, n\}$; it is Eulerian if and only if q is odd. We will give an easy constructive proof to show that G is Hamiltonian. Furthermore, we determine the independence number, chromatic number and clique number of G ; see Section 2.

Note that in graph theory literature, it is common to write $G = (V, E)$ with V as the vertex set, and E as the edge set consisting of all the unordered pairs of vertices u and v that are adjacent.

Recall that for two given graphs $G = (V, \sim)$ and $G' = (V', \sim')$, a map $\phi : V \rightarrow V'$ is a *graph homomorphism* if

$$\phi(a) \sim' \phi(b) \quad \text{in } G' \quad \text{whenever } a \sim b \text{ in } G.$$

A graph homomorphism is called a *graph endomorphism* if $G = G'$. Thus, the fundamental theorem of geometry of $\mathbb{F}_q^{m \times n}$ can be formulated in terms of graph endomorphisms on $(\mathbb{F}_q^{m \times n}, \sim)$. In [16,17], the author characterized the graph endomorphisms on symmetric matrix graphs and hermitian matrix graphs over a finite field. In Section 3, we will characterize graph endomorphisms on matrix graphs using results in Section 2.

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