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Extended Bernoulli and Stirling matrices and related combinatorial identities ☆

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ABSTRACT

In this paper we establish plenty of number theoretic and combinatoric identities involving generalized Bernoulli polynomials and Stirling numbers of both kinds, which generalize various known identities. These formulas are deduced from Pascal type matrix representations of Bernoulli and Stirling numbers. For this we define and factorize a modified Pascal matrix corresponding to Bernoulli and Stirling cases.

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1. Introduction

Matrices and matrix theory are recently used in number theory and combinatorics. In particular Pascal type lower-triangular matrices are studied with Fibonacci, Bernoulli, Stirling and Pell numbers and other special numbers sequences. Cheon and Kim [13] factorized (generalized) Stirling matrices by Pascal matrices and obtained some combinatorial identities. Zhang and Wang [31] gave product

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formulas for the Bernoulli matrix and established several identities involving Fibonacci numbers, Bernoulli numbers and polynomials.

In this paper we employ matrices for degenerate Bernoulli polynomials and generalized Stirling numbers. We define degenerate Bernoulli and generalized Stirling matrices which generalize previous results and lead some new combinatorial identities. Some of these identities can hardly be obtained by classical ways such as by using generating functions or counting, however they are easily come up via matrix representations after elementary matrix multiplication.

The summary by sections is as follows: In Section 2, we define Pascal functional matrix which is a special case of Pascal functional matrices defined in [26,32] and factorize by the summation matrices. In Section 3, we generalize Bernoulli matrix and investigate some properties. In Section 4, we define two types generalized Stirling matrices and obtain relationships between Bernoulli matrices and Stirling matrices of the second type. Furthermore, degenerate Bernoulli and generalized Stirling matrices are factorized by Pascal matrices and several identities are developed as a result of matrix representations. In final section, we introduce some special cases of the results obtained in Section 4.

Throughout this paper we assume that i, j and n are nonnegative integers; μ, λ, w and x are real or complex numbers.

2. Pascal matrix

Let $g(t)$ be a formal power series of the form

$$g(t)=\sum_{m=0}^{\infty} g_m \frac{t^m}{m!}.$$

Define the multiplication matrix $M(g)$ as the lower triangular matrix whose (i,j) entry is $g_{i-j}/(i-j)!$. The map $g \to M(g)$ is an algebra isomorphism from the formal power series to the lower triangular Toeplitz matrices [18, Chapter 1]. Now define the diagonal matrix $F=\operatorname{diag}(0!,1!,2!,\ldots)$ and the Pascal matrix associated with $g(t)$ by $P(g)=FM(g)F^{-1}$. It is obvious that the set of all such Pascal matrices is isomorphic to the algebra of lower triangular Toeplitz matrices. These matrices satisfy

$$P(g)P(f)=P(gf), \qquad P(g)^k=P\left(g^k\right) \quad \text{and} \quad P(g)^{-1}=P(1/g) \quad \text{when } g_0 \neq 0. \tag{1}$$

The $n \times n$ section of an infinite matrix $P(g)$ is defined as the finite submatrix composed of the first n rows and columns of $P(g)$. Also, (1) is valid for the $n \times n$ sections.

Let $\mathcal{P}_n[\lambda,x]$ be the $n \times n$ section of the infinite Pascal matrix $P(g)$ associated with the generating function

$$g(t)=(1+\lambda t)^{x/\lambda}=\sum_{m=0}^{\infty}(x|\lambda)_m \frac{t^m}{m!},$$

i.e., let $\mathcal{P}_n[\lambda,x]$ be the $n \times n$ matrix defined by

$$\left(\mathcal{P}_n[\lambda,x]\right)_{i,j}=\begin{cases}\binom{i-1}{j-1}(x|\lambda)_{i-j}, & \text{if } i \geqslant j \geqslant 1,\\ 0, & \text{if } 1 \leqslant i < j,\end{cases}$$

where $(x|\lambda)_k=x(x-\lambda)(x-2\lambda)\cdots(x-(k-1)\lambda)$ with $(x|\lambda)_0=1$.

From (1) we have

$$\mathcal{P}_n[\lambda,x+y]=\mathcal{P}_n[\lambda,x]\mathcal{P}_n[\lambda,y], \qquad \left(\mathcal{P}_n[\lambda,x]\right)^h=\mathcal{P}_n[\lambda,hx] \quad \text{and} \quad \mathcal{P}_n^{-1}[\lambda,x]=\mathcal{P}_n[\lambda,-x].$$

The algebraic properties of $\mathcal{P}_n[\lambda,x]$ can be found in [3,7,16,26,29,30,32]. In fact, $\mathcal{P}_n[-\lambda,x]$ is the matrix $\mathcal{P}_{n,\lambda}[x]$ defined in [3] and this matrix is a special case of the generalized Pascal functional matrices defined in [26,32]. So we will not discuss the algebraic properties of this matrix. We will only focus on factorizing this matrix by the summation matrices. For this purpose, let us define the $n \times n$ matrices $\mathcal{R}_n[\lambda,x]$ and $G_k[\lambda,x]$ by

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