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Robustness of fuzzy interval circulant-Hankel matrices

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ABSTRACT

Fuzzy algebra is an algebraic structure in which classical addition and multiplication are replaced by $\oplus$ and $\otimes$, where $a \oplus b = \max\{a, b\}$, $a \otimes b = \min\{a, b\}$. A square matrix is circulant-Hankel if the input values in every row are the same as the values in the previous row, but they are cyclically shifted by one position to the left.

The robustness and the $\boldsymbol{X}$-robustness of fuzzy circulant-Hankel matrices are studied. We define the possible and universal robustness and $\boldsymbol{X}$-robustness of interval matrices. The necessary and sufficient conditions for the possible and universal robustness and $\boldsymbol{X}$-robustness of interval circulant-Hankel matrices are given.

1. Introduction

Studying matrix properties in fuzzy algebra, where addition and multiplication are formally replaced by the operations of maximum and minimum, is of great importance for applications in various areas. Fuzzy discrete dynamic systems, which can be introduced by fuzzy matrices, are useful for describing knowledge engineering, scheduling, cluster analysis, fuzzy logic programs [7], diagnosis of technical devices [18,19] or medical diagnosis [16]. Periodic behavior of fuzzy matrices and orbits with corresponding polynomial algorithms were studied in [3,4,17].

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In practice, matrix and vector inputs are represented by intervals of possible values rather than exact values. Considering matrices and vectors with interval coefficients is therefore of great practical importance, see [2,6,8,9,15].

In this paper we shall deal with special class of matrices, circulant-Hankel matrices. We derive the formula for computing the period and the orbit period of a given circulant-Hankel matrix. Moreover, we define the universal and possible robustness ($\boldsymbol{X}$-robustness) of interval circulant-Hankel matrices and derive the polynomial algorithms for checking them.

2. Motivation

The main reason for checking the robustness is that this property is used to describe many aspects of everyday life as mentioned in the introduction. Let us introduce the following model.

Suppose that the user U is buying some object, for example an apartment. There are several apartments available. The user selects them under properties $p_1, p_2, \dots, p_n$ (size, number of rooms, orientation, reconstruction, etc.). Let $A = (a_{ij})$ be a matrix, where a_{ij} represents a measure of the preference of the property p_i before p_j. If the user prefers the property a_i against p_j, he puts as great value a_{ij} into the matrix A as his preference is. Let I corresponds to the maximum preference and O in r_{ij} symbolizes that user does not give priority to the property p_i before p_j. Each object has several attributes. Let x be the vector of initial interest, where $x_i \in [O, I]$ represents the degree of the preference of the property p_i. The vector x is modified according to the matrix A to the vectors of interest $A \otimes x$, $A^2 \otimes x, \dots,$ where $\otimes$ and $\oplus$ are binary operations minimum and maximum, respectively. We are looking for the vector of interest on which A produces no effect, i.e., the maximum stable vector of the matrix $A = (a_{ij})$ in the form $A^k \otimes x$ for some natural number k. The preference of each property in those vector is reflected in the summary aggregation function, which allows to order the proposed apartments according to the value of this function.

Since we require the existence of the stable vector for each vector x, therefore, the robustness is a necessary condition for the preference matrix A. In this paper we shall deal with the robustness of the special class of matrices, circulant-Hankel matrices.

3. Preliminaries

The fuzzy algebra $\mathcal{B}$ is the triplet $(B, \oplus, \otimes)$, where $(B, \leqslant)$ is a bounded linearly ordered set with binary operations *maximum* and *minimum*, denoted by $\oplus$ and $\otimes$, respectively. The least element in B will be denoted by O, the greatest one by I.

By $\mathbb{N}$ we denote the set of all natural numbers and by $\mathbb{N}_0$ the set $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$. The greatest common divisor of a set $S \subseteq \mathbb{N}$ is denoted by $\gcd S$ and the least common multiple by $\operatorname{lcm} S$. For a given natural number $n \in \mathbb{N}$, we shall use the notations $N = \{1, 2, \dots, n\}$ and $N_0 = \{0, 1, \dots, n-1\}$.

For any $n \in \mathbb{N}$, $B(n, n)$ denotes the set of all square matrices of order n and $B(n)$ denotes the set of all n-dimensional column vectors over B. The matrix operations over $\mathcal{B}$ are defined formally in the same manner (with respect to $\oplus$, $\otimes$) as matrix operations over any field. The r-th power of a matrix A is denoted by A^r, with elements $(A^r)_{ij}$.

For $A \in B(n, n)$, $C \in B(n, n)$ we write $A \leqslant C$ $(A < C)$ if $a_{ij} \leqslant c_{ij}$ $(a_{ij} < c_{ij})$ holds true for all $i, j \in N$.

By digraph we understand a pair $\mathcal{G} = (V_{\mathcal{G}}, E_{\mathcal{G}})$, where $V_{\mathcal{G}}$ is a non-empty finite set, called the node set, and $E_{\mathcal{G}} \subseteq V_{\mathcal{G}} \times V_{\mathcal{G}}$, called the edge set. A digraph $\mathcal{G}' = (V_{\mathcal{G}'}, E_{\mathcal{G}'})$ is a subdigraph of digraph $\mathcal{G}$, if $V_{\mathcal{G}'} \subseteq V_{\mathcal{G}}$ and $E_{\mathcal{G}'} \subseteq E_{\mathcal{G}}$. Specially, $\mathcal{G}/V_{\mathcal{G}'}$ stands for the subdigraph of $\mathcal{G}$ induced by the vertex set $V_{\mathcal{G}'}$, i.e., $V_{\mathcal{G}/V_{\mathcal{G}'}} = V_{\mathcal{G}'}$ and $E_{\mathcal{G}/V_{\mathcal{G}'}} = \{(i, j) \in E_{\mathcal{G}};\ i, j \in V_{\mathcal{G}'}\}$.

A path in a digraph $\mathcal{G}$ is the sequence $\mathcal{P} = (v_0, e_1, v_1, e_2, v_2, \dots, v_{l-1}, e_l, v_l)$ of nodes and edges such that $e_k = (v_{k-1}, v_k) \in E_{\mathcal{G}}$ for $k = 1, 2, \dots, l$. The number l is the length of the path $\mathcal{P}$ and is denoted by $\ell(\mathcal{P})$. If $v_0 = v_l$, then $\mathcal{P}$ is called a cycle. A cycle is elementary if all nodes except the terminal node are distinct. A digraph is acyclic if there is no cycle of positive length in $\mathcal{G}$. A digraph is called strongly connected if any two distinct nodes of $\mathcal{G}$ are contained in a common cycle. By a strongly connected component of $\mathcal{G}$ we mean a maximal strongly connected subdigraph of $\mathcal{G}$. A strongly connected component $\mathcal{K} = (V_{\mathcal{K}}, E_{\mathcal{K}})$ is called non-trivial if there is a cycle of positive length in $\mathcal{K}$. By $\mathrm{SCC}^{\star}\mathcal{G}$ we denote the set of all non-trivial strongly connected components of $\mathcal{G}$. For