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Left-looking version of *AINV* preconditioner with complete pivoting strategy

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ABSTRACT

In this paper, we apply a complete pivoting strategy to compute the left-looking version of *AINV* preconditioner for linear systems. As the preprocessing, the MultiLevel Nested Dissection reordering has also been applied. We have used this preconditioner as the right preconditioner for several linear systems where the coefficient matrices have been downloaded from the University of Florida Sparse Matrix Collection. Numerical experiments presented in this paper indicate the effectiveness of such a complete pivoting on the quality of left-looking version of *AINV* preconditioner.

1. Introduction

Krylov subspace methods [9] are examples of iterative methods to solve the linear system of equations of the form

$$Ax = b, \tag{1}$$

where the coefficient matrix $A \in \mathbb{R}^{n\times n}$ is nonsingular, large, sparse and nonsymmetric and also $x, b \in \mathbb{R}^n$. A good preconditioner will accelerate the solution of this system.

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An explicit preconditioner M for system (1) is an approximation of matrix A^{-1}, i.e., $M \approx A^{-1}$. This preconditioner will change the original system (1) to the right preconditioned system

$$AMu = b; \qquad Mu = x, \tag{2}$$

and then, the Krylov subspace methods are applied to solve system (2). The most well-known explicit preconditioner is the *AINV* preconditioner [1]. There are two left and right-looking versions for this preconditioner. The left and right-looking versions of this preconditioner are computed when one applies dropping in the left and right-looking versions of A-biconjugation process, respectively. Suppose that matrix A has the

$$A = \bar{L}\bar{D}\bar{U}, \tag{3}$$

factorization where $\bar{L}$ and $\bar{U}^T$ are unit lower triangular matrices and $\bar{D}$ is a diagonal matrix. *IJK* version of Gaussian Elimination is an algorithm to compute this factorization [9]. This process works with the Schur-Complement matrix, explicitly and computes $\bar{L}$ and $\bar{U}$ factors column-wise and row-wise, respectively.

Based on the connection between the right-looking version of A-biconjugation process and the *IJK* version of Gaussian Elimination, Bollhöfer and Saad have presented a complete pivoting strategy for the right-looking version of *AINV* preconditioner [3]. In this paper, we extend pivoting for the left-looking version of this preconditioner and consider the effectiveness of this pivoting. This extension is based on the connection between the left-looking version of A-biconjugation process and the *IJK* version of Gaussian Elimination. Left-looking version of A-biconjugation process does not deal with the Schur-Complement matrix, explicitly, but it enables us to generate special parts of this matrix, implicitly. We have used the pivoting protocol of the *IJK* version of Gaussian Elimination as a navigator to apply pivoting in the left-looking version of A-biconjugation process.

In this paper, notation $X_{i:j,k}$ indicates the entries of the k-th column of matrix X whose row indices are between i and j. We have also used $X_{k,i:j}$ to show the entries of the k-th row of matrix X whose column indices are between i and j. $X_{:,k}$ and $X_{k,:}$ are considered as the k-th column and the k-th row of matrix X, respectively. In this paper, the term *MLND* reordering refers to the MultiLevel Nested Dissection reordering.

In Section 2 of this paper, we study the relation between the left-looking version of A-biconjugation process and the *IJK* version of Gaussian Elimination. In Section 3, the left-looking version of *AINV* preconditioner, which is computed by using a complete pivoting strategy, has been presented. In Section 4, numerical experiments are proposed and implementation details are discussed.

2. Relation between the left-looking version of *A*-biconjugation process and the *IJK* version of Gaussian Elimination

Algorithm 1, computes the factorization (3) and is termed the *IJK* version of Gaussian Elimination [4,8]. At the beginning of step i of this algorithm, the relation

$$A=\underbrace{\begin{bmatrix}1&&&&&&\\&1&&&&&\\&&\ddots&&&&\\&&&1&&&\\\bar g_1&\bar g_2&&\bar g_{i-1}&1&&\\&&&&&\ddots&\\&&&&&&1\end{bmatrix}}_{\bar L}\underbrace{\begin{bmatrix}\bar d_{11}&&&\\&\ddots&&\\&&\bar d_{i-1,i-1}&\\&&&(\bar S^{(i-1)})_{j,k\geqslant i}\end{bmatrix}}_{\bar S^{(i-1)}}\underbrace{\begin{bmatrix}1&&&&\bar h_1&&\\&1&&&\bar h_2&&\\&&\ddots&&&&\\&&&1&\bar h_{i-1}&&\\&&&&1&&\\&&&&&\ddots&\\&&&&&&1\end{bmatrix}}_{\bar U}, \tag{4}$$

holds where $\bar g_k \in \mathbb{R}^{(n-k)\times 1}$ and $\bar h_k \in \mathbb{R}^{1\times(n-k)}$, for $1 \leqslant k \leqslant i-1$, are the already computed columns and rows of the matrices $\bar L$ and $\bar U$, respectively and the submatrix $(\bar S^{(i-1)})_{j,k\geqslant i}$ is available. At the end of step i of this algorithm, matrix $\bar S^{(i-1)}$ will change to a new matrix which we term it $\bar S^{(i)}$. At