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Consensus of high-order multi-agent systems with switching topologies

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ABSTRACT

In this paper, a consensus problem is investigated for high-order multi-agent systems with switching communication networks, through which only output information instead of full-state information can be transmitted to neighbors. Based on self-state-feedback and neighbor-output-feedback, a new consensus protocol is proposed, which can realize arbitrary convergence rate. Furthermore, as an application, a nonlinear heading consensus protocol is designed for a multi-vehicle model. Finally, numerical simulations are given to illustrate the theoretical results.

1. Introduction

Consensus or synchronization phenomena are widespread in the real world such as the synchronized motion of pendulum clocks, synchronous flashing of fireflies and the rhythmic behavior of pacemaker cells in the heart [1]. For a theoretical explanation to some consensus phenomena, many elegant mathematical models were proposed, for example, Boid model [2], Vicsek model [3] and Kuramoto model [4]. In the past decades, the consensus idea and those models have brought great influence on many disciplines including mathematics [5–7], physics [3,8], biology [9], computer science [2,10], engineering [11,12], psychology [13] and so on. The consensus problem of the so-called multi-agent systems has been paid great attention by the community of control theory since the pioneering contributions including [14–20]. For linear multi-agent systems, if the interconnection topologies of the networks are fixed, a direct method solving the consensus problem is to analyze the eigenvalues of the closed-loop systems. But this method is invalid for the case of switching topologies

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since the closed-loop systems are switched systems whose dynamical behavior cannot be determined by the eigenvalues [21–26]. For discrete multi-agent systems, there are many approaches to deal with switching topologies such as the infinite products of stochastic matrices [14,27], the reduction to absurdity [28], the graphical method [29]. Different from the discrete case, for continuous multi-agent systems, the main methods to deal with switching topologies lie in the common Lyapunov function method [15,30–32], reducing to the discrete case [18] and the system transformation method [33–35]. For more reviews on switching topologies, we refer to the recent survey [12]. Moreover, it is definitely worth noting that the topic of synchronization of complex networks is closely related to the consensus of multi-agent systems [36]. There have been a lot of contributions on the synchronization of switching networks especially stochastic networks [38]. Some important stochastic switching networks are investigated such as small-world networks [37,38], blinking networks [39,40], moving neighborhood networks [41,42] and numerosity-constrained networks [43]. Some important properties on switching networks like the convergence speed [44] and the robustness with respect to link failures and channel noise [45] are also concerned.

Here, we focus on the system transformation method proposed in [33]. For a class of protocols of the second-order multi-agent systems, through a system transformation, the closed-loop systems can be transformed to the first-order multi-agent systems considered by [15] and [18]. It has been shown that the system transformation method is effective to many kinds of multi-agent systems with switching topologies [33–35,46]. In our recent paper [47], the model transformation method is modified to realize arbitrary convergence rate, which plays an important role in analyzing the flocking of multi-agent systems with proximity graphs. In [48] and [49], the model transformation method is used for high-order discrete and continuous multi-agent systems respectively. However, for the case of switching topology, how to design a high-order protocol to realize an arbitrary convergence rate is still an open problem.

In this paper, for high-order multi-agent systems with switching topologies, a new consensus protocol based on self-state-feedback and neighbor-output-feedback is designed and an arbitrary convergence rate can be realized by adjusting a control gain. Furthermore, as an application, a nonlinear protocol is designed to solve the heading consensus problem of a class of multi-vehicle systems. Finally, numerical simulations are given to illustrate the theoretical results.

2. Preliminaries and problem statement

Consider the multi-agent system with switching topology $\mathcal{G}_t = (\mathcal{V}, \mathcal{E}_t, \mathcal{A}_t)$, which is a weighted digraph with the set of nodes $\mathcal{V} = \{1, 2, \dots, m\}$, set of edges $\mathcal{E}_t \subset \mathcal{V} \times \mathcal{V}$, and a non-negative adjacency matrix $\mathcal{A}_t = (a_{ij}(t))$. An edge of $\mathcal{G}_t$ is denoted by (i, j), which means node j can receive the output information from i. Adjacency matrix $\mathcal{A}_t$ is defined such that $a_{ij}(t) > 0$ if $(j, i) \in \mathcal{E}_t$, while $a_{ij}(t) = 0$ if $(j, i) \notin \mathcal{E}_t$. If $(j, i) \in \mathcal{E}_t$, we say j is a *neighbor* of i. We denote the set of neighbors of node i by $N_i(t)$. The Laplacian matrix of the weighted digraph is defined as $L_t = (l_{ij}(t))$, where $l_{ii}(t) = \sum_{j \neq i} a_{ij}(t)$ and $l_{ij}(t) = -a_{ij}(t)$ $(i \neq j)$. Let $\mathbf{1}_n$ denote the $n \times 1$ column vector of all ones. It is obvious that $L_t \mathbf{1}_n = 0$. Let I_n denote the $n \times n$ identity matrix. A digraph is called *strongly connected* if and only if any two distinct nodes of the graph can be connected via a path that follows the direction of the edges of the digraph; while it is called *weakly connected* if replacing all of its directed edges with undirected edges produces a connected graph. The *mirror graph* of $\mathcal{G}_t$ is an undirected graph with the same set of nodes as $\mathcal{G}_t$ and the symmetric adjacency matrix $\hat{A}_t$ with entries $\hat{a}_{ij} = \hat{a}_{ji} = (a_{ij} + a_{ji})/2$. A digraph is called *balanced* if the in-degree and the out-degree of each i are equal, i.e.,

$$\deg_{\text{in}}(i) := \sum_{j=1}^{n} a_{ij} = \sum_{j=1}^{n} a_{ji} := \deg_{\text{out}}(i).$$

For a balanced digraph $\mathcal{G}$, by Lemma 2.6.1 of [50], we know that $\mathcal{G}$ is weakly connected if and only if $\mathcal{G}$ is strongly connected. Thus we prefer to call $\mathcal{G}$ a *connected balanced digraph* if it is weakly connected and balanced.

Suppose each vertex of digraph $\mathcal{G}_t$ is a dynamic agent with the dynamics described by

$$\dot{x}_i = f(x_i) + g(x_i)u_i, \qquad y_i = h(x_i),$$