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On the Laplacian-energy-like invariant

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ABSTRACT

Let G be a connected graph of order n with Laplacian eigenvalues $\mu_1 \geqslant \mu_2 \geqslant \cdots \geqslant \mu_{n-1} > \mu_n = 0$. The Laplacian-energy-like invariant of the graph G is defined as

$$LEL = LEL(G) = \sum_{i=1}^{n-1} \sqrt{\mu_i}.$$

Lower and upper bounds for LEL are obtained, in terms of n, number of edges, maximum vertex degree, and number of spanning trees.
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1. Introduction

Let $G = (V, E)$ be a simple undirected graph with vertex set $V(G) = \{v_1, v_2, \ldots, v_n\}$ and edge set $E(G)$, $|E(G)| = m$. Let d_i be the degree of the vertex v_i for $i = 1, 2, \ldots, n$. The maximum vertex degree is denoted by Δ.

Let $\mathbf{A}(G)$ be the $(0, 1)$-adjacency matrix of G and $\mathbf{D}(G)$ be the diagonal matrix of vertex degrees. The Laplacian matrix of G is $\mathbf{L}(G) = \mathbf{D}(G) - \mathbf{A}(G)$. This matrix has nonnegative eigenvalues $n \geqslant \mu_1 \geqslant \mu_2 \geqslant \cdots \geqslant \mu_n = 0$. Denote by $Spec(G) = \{\mu_1, \mu_2, \ldots, \mu_n\}$ the spectrum of $\mathbf{L}(G)$, i.e., the Laplacian spectrum of G. When more than one graph is under consideration, then we write $\mu_i(G)$ instead of μ_i.

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As well known [17], a graph of order n has

$$t = t(G) = \frac{1}{n} \prod_{i=1}^{n-1} \mu_i \tag{1}$$

spanning trees and

$$\sum_{i=1}^{n} \mu_i = 2m. \tag{2}$$

In 2008 Liu and Liu [16] considered a new Laplacian-spectrum-based graph invariant

$$LEL = LEL(G) = \sum_{k=1}^{n-1} \sqrt{\mu_k} \tag{3}$$

and named it *Laplacian-energy-like invariant*. The motivation for introducing *LEL* was in its analogy [10] to the earlier much studied graph energy [6,9,14]. For details on *LEL* see the review [15], the recent papers [3,10,11,22,20,18,7,5,19], and the references cited therein. For previously established bounds on *LEL* see in [15,7]. Of these we mention the simple estimates [8,21,22]

$$\sqrt{2m} \leqslant LEL \leqslant \sqrt{2m(n-1)}$$

and their recent improvement [7]

$$\sqrt{\frac{4m(n-1)}{n} + (n-1)(n-2)(n\,t)^{1/(n-1)}} \leqslant LEL \leqslant \sqrt{\frac{2m(n-1)^2}{n} + (n\,t)^{1/(n-1)}}\,. \tag{4}$$

In what follows, we obtain a few more lower and upper bounds on *LEL* in terms of n, m, Δ, and t.

As usual, K_n, P_n, $K_{1,\,n-1}$, and $K_{p,\,q}$ $(p+q=n)$ denote, respectively, the complete graph, the path, the star, and a complete bipartite graph on n vertices.

2. Bounds on Laplacian-energy-like invariant

In order to arrive at one of our main results, we need four previously known lemmas.

Lemma 2.1. [17] Let G be a graph on n vertices with at least one edge. Then

$$\mu_1 \geqslant \Delta + 1\,. \tag{5}$$

Moreover, if G is connected, then the equality in (5) holds if and only if $\Delta = n-1$.

Lemma 2.2. [17] Let G be a graph of order n and $\overline{G}$ its complement. If $Spec(G) = \{\mu_1, \mu_2, \ldots, \mu_{n-1}, 0\}$, then $Spec(\overline{G}) = \{n-\mu_1, n-\mu_2, \ldots, n-\mu_{n-1}, 0\}$. From this, it follows that $\mu_1(G) \leqslant n$ with equality holding if and only if $\overline{G}$ is disconnected.

Lemma 2.3. [2] Let G be a connected graph with $n \geqslant 3$ vertices. Then $\mu_2 = \mu_3 = \cdots = \mu_{n-1}$ if and only if $G \cong K_n$ or $G \cong K_{1,n-1}$ or $G \cong K_{\Delta,\Delta}$.

Lemma 2.4. [13] Let $x_1, x_2, \ldots, x_N$ be non-negative numbers, and let

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