

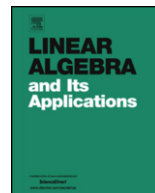


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## On the Laplacian-energy-like invariant



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### ABSTRACT

Let  $G$  be a connected graph of order  $n$  with Laplacian eigenvalues  $\mu_1 \geq \mu_2 \geq \dots \geq \mu_{n-1} > \mu_n = 0$ . The Laplacian-energy-like invariant of the graph  $G$  is defined as

$$LEL = LEL(G) = \sum_{i=1}^{n-1} \sqrt{\mu_i}.$$

Lower and upper bounds for  $LEL$  are obtained, in terms of  $n$ , number of edges, maximum vertex degree, and number of spanning trees.

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### 1. Introduction

Let  $G = (V, E)$  be a simple undirected graph with vertex set  $V(G) = \{v_1, v_2, \dots, v_n\}$  and edge set  $E(G)$ ,  $|E(G)| = m$ . Let  $d_i$  be the degree of the vertex  $v_i$  for  $i = 1, 2, \dots, n$ . The maximum vertex degree is denoted by  $\Delta$ .

Let  $\mathbf{A}(G)$  be the  $(0, 1)$ -adjacency matrix of  $G$  and  $\mathbf{D}(G)$  be the diagonal matrix of vertex degrees. The Laplacian matrix of  $G$  is  $\mathbf{L}(G) = \mathbf{D}(G) - \mathbf{A}(G)$ . This matrix has nonnegative eigenvalues  $n \geq \mu_1 \geq \mu_2 \geq \dots \geq \mu_n = 0$ . Denote by  $\text{Spec}(G) = \{\mu_1, \mu_2, \dots, \mu_n\}$  the spectrum of  $\mathbf{L}(G)$ , i.e., the Laplacian spectrum of  $G$ . When more than one graph is under consideration, then we write  $\mu_i(G)$  instead of  $\mu_i$ .

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As well known [17], a graph of order  $n$  has

$$t = t(G) = \frac{1}{n} \prod_{i=1}^{n-1} \mu_i \tag{1}$$

spanning trees and

$$\sum_{i=1}^n \mu_i = 2m. \tag{2}$$

In 2008 Liu and Liu [16] considered a new Laplacian-spectrum-based graph invariant

$$LEL = LEL(G) = \sum_{k=1}^{n-1} \sqrt{\mu_k} \tag{3}$$

and named it *Laplacian-energy-like invariant*. The motivation for introducing *LEL* was in its analogy [10] to the earlier much studied graph energy [6,9,14]. For details on *LEL* see the review [15], the recent papers [3,10,11,22,20,18,7,5,19], and the references cited therein. For previously established bounds on *LEL* see in [15,7]. Of these we mention the simple estimates [8,21,22]

$$\sqrt{2m} \leq LEL \leq \sqrt{2m(n-1)}$$

and their recent improvement [7]

$$\sqrt{\frac{4m(n-1)}{n} + (n-1)(n-2)(nt)^{1/(n-1)}} \leq LEL \leq \sqrt{\frac{2m(n-1)^2}{n} + (nt)^{1/(n-1)}}. \tag{4}$$

In what follows, we obtain a few more lower and upper bounds on *LEL* in terms of  $n, m, \Delta$ , and  $t$ .

As usual,  $K_n, P_n, K_{1, n-1}$ , and  $K_{p, q}$  ( $p + q = n$ ) denote, respectively, the complete graph, the path, the star, and a complete bipartite graph on  $n$  vertices.

## 2. Bounds on Laplacian-energy-like invariant

In order to arrive at one of our main results, we need four previously known lemmas.

**Lemma 2.1.** [17] Let  $G$  be a graph on  $n$  vertices with at least one edge. Then

$$\mu_1 \geq \Delta + 1. \tag{5}$$

Moreover, if  $G$  is connected, then the equality in (5) holds if and only if  $\Delta = n - 1$ .

**Lemma 2.2.** [17] Let  $G$  be a graph of order  $n$  and  $\bar{G}$  its complement. If  $Spec(G) = \{\mu_1, \mu_2, \dots, \mu_{n-1}, 0\}$ , then  $Spec(\bar{G}) = \{n - \mu_1, n - \mu_2, \dots, n - \mu_{n-1}, 0\}$ . From this, it follows that  $\mu_1(G) \leq n$  with equality holding if and only if  $\bar{G}$  is disconnected.

**Lemma 2.3.** [2] Let  $G$  be a connected graph with  $n \geq 3$  vertices. Then  $\mu_2 = \mu_3 = \dots = \mu_{n-1}$  if and only if  $G \cong K_n$  or  $G \cong K_{1, n-1}$  or  $G \cong K_{\Delta, \Delta}$ .

**Lemma 2.4.** [13] Let  $x_1, x_2, \dots, x_N$  be non-negative numbers, and let

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