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Representation dimensions of triangular matrix algebras ☆

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ABSTRACT

Let A be a finite dimensional hereditary algebra over an algebraically closed field k, $T_2(A) = \begin{pmatrix} A & 0 \\ A & A \end{pmatrix}$ be the triangular matrix algebra. We prove that rep.dim $T_2(A)$ is at most three if A is Dynkin type and rep.dim $T_2(A)$ is at most four if A is not Dynkin type.

Let $A^{(1)} = \begin{pmatrix} A & 0 \\ DA & A \end{pmatrix}$ be the duplicated algebra of A. Let T be a tilting A-module and $\overline{T} = T \oplus \overline{P}$ be a tilting $A^{(1)}$-module. We show that $\mathrm{End}_{A^{(1)}} \overline{T}$ is representation finite if and only if the full subcategory $\{(X_A, Y_A, f) \mid X_A \in \mathrm{mod}\, A, Y_A \in \tau^{-1}\mathscr{F}(T_A) \cup \mathrm{add}\, A\}$ of mod $T_2(A)$ is of finite type, where τ is the Auslander–Reiten translation and $\mathscr{F}(T_A)$ is the torsion-free class of mod A associated with T.

Moreover, we also prove that rep.dim $\mathrm{End}_{A^{(1)}} \overline{T}$ is at most three if A is Dynkin type.

1. Introduction

Representation dimension of Artin algebras was introduced by Auslander in [4], this concept gives a reasonable way of measuring how far an Artin algebra Λ is from being representation-finite. In particular, Auslander has shown that an Artin algebra is representation-finite if and only if its representation dimension is at most 2.

Iyama, in [14], proved that the representation dimension of an Artin algebra is always finite. Recently, Rouquier proved in [19] that the representation dimension of Artin algebras can be arbitrary large.

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An interesting relationship between the representation dimension and the finitistic dimension conjecture has been shown by Igusa and Todorov [13], that is, if the representation dimension of an algebra is at most three, then its finitistic dimension is finite. Since then, many important algebras were proved to have representation dimension at most three, such as tilted algebras, m-replicated algebras, quasi-tilted algebras etc., see [2,16,17] for details.

We follow this direction and investigate some special kinds of triangular matrix algebras with small representation dimensions.

Let A be a finite dimensional hereditary algebra over an algebraically closed field k, and $T_2(A) = \begin{pmatrix} A & 0 \\ A & A \end{pmatrix}$ be the triangular matrix algebra.

The following theorem is one of the main results of this paper.

Theorem 1. *Let A be a finite dimensional hereditary algebra over an algebraically closed field k. Then* rep.dim $T_2(A) \leqslant 3$ *if A is Dynkin type and* $3 \leqslant$ rep.dim $T_2(A) \leqslant 4$ *if A is not Dynkin type.*

Remark. Theorem 1 improves the well known result about representation dimension of $T_2(A)$. According to [9], we know that rep.dim $T_2(A) \leqslant$ rep.dim $A + 2$, which implies that rep.dim $T_2(A) \leqslant 5$ if A is a finite dimensional hereditary algebra over an algebraically closed field.

Let A be a finite dimensional hereditary algebra over an algebraically closed field k, and let $A^{(1)} = \begin{pmatrix} A & 0 \\ DA & A \end{pmatrix}$ be the duplicated algebra of A. Tilting theory of duplicated algebra $A^{(1)}$ has strong relationship with cluster tilting theory introduced in [7], and it has been widely investigated in [1,15,21,22]. In the second part of this paper, we mainly investigate the representation type and representation dimension of endomorphism algebras of tilting modules over duplicated algebra $A^{(1)}$.

Theorem 2. *Let A be a finite dimensional hereditary algebra over an algebraically closed field k, and $A^{(1)}$ be the duplicated algebra of A. Let T be a basic tilting A-module and $\overline{T} = T \oplus \overline{P}$ be a tilting $A^{(1)}$-module, where $\overline{P}$ is the direct sum of all non-isomorphic indecomposable projective-injective $A^{(1)}$-modules. Then* $\mathrm{End}_{A^{(1)}} \overline{T}$ *is representation finite if and only if the full subcategory $\{(X_A, Y_A, f) \mid X_A \in \mathrm{mod}A,\ Y_A \in \tau^{-1}\mathscr{F}(T_A) \cup \mathrm{add}\,A\}$ of* mod $T_2(A)$ *is of finite type, where τ is the Auslander–Reiten translation and $\mathscr{F}(T_A)$ is the torsion-free class associated with T.*

Theorem 3. *Let A be a finite dimensional hereditary algebra over an algebraically closed field k, and $A^{(1)}$ be the duplicated algebra of A. Let T be a basic tilting A-module and $\overline{T} = T \oplus \overline{P}$ be a tilting $A^{(1)}$-module, where $\overline{P}$ is the direct sum of all non-isomorphic indecomposable projective-injective $A^{(1)}$-modules. If A is Dynkin type, then* rep.dim $\mathrm{End}_{A^{(1)}} \overline{T} \leqslant 3$.

Remark. We should mention that Theorem 1 can be obtained from Theorem 3 by taking $T = DA$. We prove them differently, which seems to be of independent interest.

This paper is arranged as follows. In Section 2, we collect definitions and basic facts needed for our research. Section 3 is devoted to the proof of Theorem 1, and in Section 4, we prove Theorems 2 and 3.

2. Preliminaries

Let Λ be a finite dimensional algebra over an algebraically closed field k. We denote by mod Λ the category of all finitely generated right Λ-modules and by ind Λ the full subcategory of mod Λ containing exactly one representative of each isomorphism class of indecomposable Λ-modules. We denote by pd X (resp. id X) the projective (resp. injective) dimension of a Λ-module X and by gl.dim Λ the global dimension of Λ. Let $D = \mathrm{Hom}_k(-,\ k)$ be the standard duality between mod Λ and mod