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On backward errors of structured polynomial eigenproblems solved by structure preserving linearizations

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ABSTRACT

We derive explicit computable expressions of structured backward errors of approximate eigenelements of structured matrix polynomials including symmetric, skew-symmetric, Hermitian, skew-Hermitian, even and odd polynomials. We determine minimal structured perturbations for which approximate eigenelements are exact eigenelements of the perturbed polynomials. We also analyze structured pseudospectra of a structured matrix polynomial and establish a partial equality between unstructured and structured pseudospectra. Finally, we analyze the effect of structure preserving linearizations of structured matrix polynomials on the structured backward errors of approximate eigenelements and show that structure preserving linearizations which minimize structured condition numbers of eigenvalues also minimize the structured backward errors of approximate eigenelements.

1. Introduction

Consider a matrix polynomial $\mathrm{P}(z) := \sum_{j=0}^{m} z^j A_j$ of degree m, where $A_j \in \mathbb{C}^{n\times n}$. We assume that P is regular, that is, $\det(\mathrm{P}(z)) \neq 0$ for some $z \in \mathbb{C}$. We say that $\lambda \in \mathbb{C}$ is an eigenvalue of P if $\det(\mathrm{P}(\lambda)) = 0$. A nonzero vector $x \in \mathbb{C}^n$ (respectively, $y \in \mathbb{C}^n$) that satisfies $\mathrm{P}(\lambda)x = 0$ (respectively, $y^H\mathrm{P}(\lambda) = 0$) is called a right (respectively, left) eigenvector of P corresponding to the eigenvalue λ. The standard approach to computing eigenelements of P is to convert P into an equivalent linear polynomial L, called a linearization of P, and employ a numerically backward stable algorithm to compute the eigenelements of L, where $\mathrm{L}(z) := zX + Y$, $X \in \mathbb{C}^{mn\times mn}$ and $Y \in \mathbb{C}^{mn\times mn}$. However, a

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matrix polynomial admits many linearizations. In fact, two vector spaces of potential linearizations of a matrix polynomial P have been identified and analyzed in [20,18]. Thus choosing an optimal (in some sense) linearization of P is an important first step towards computing eigenelements of P. In general, a linearization of P can have an adverse effect on the conditioning of the eigenvalues of P (see [13]). Hence by analyzing the condition numbers of eigenvalues of linearizations, potential linearizations of P have been identified in [13] whose eigenvalues are almost as sensitive to perturbations as that of P. Further, it has been shown in [11] that these linearizations nearly minimize the backward errors of approximate eigenelements.

Polynomial eigenvalue problems that occur in many applications possess some distinctive structures (e.g., Hermitian, even, odd and palindromic) which in turn induce certain spectral symmetries on the eigenvalues of the matrix polynomials (see [21,25,24,17,16] and the references therein). With a view to preserving spectral symmetry in the computed eigenvalues (and possibly improved accuracy), there has been a lot of interests in developing structured preserving algorithms (see [15,23,25,19,9] and the references therein). Since linearization is the standard way to solve a polynomial eigenvalue problem, for a structured matrix polynomial it is therefore desirable to choose a structured linearization and then solve the linear problem by a backward stable structure preserving algorithm. For the accuracy assessment of computed solution, it is therefore important to understand the sensitivity of eigenvalues of a structured matrix polynomial with respect to structure preserving perturbations. Also it is equally important to know the structured backward errors of approximate eigenelements of a structured matrix polynomial. Moreover, for a variety of structured polynomials such as symmetric, skew-symmetric, Hermitian, skew-Hermitian, even, odd and palindromic polynomials, there are infinitely many structured linearizations, see [12,21]. This poses a genuine problem of choosing one linearization over the other. For computational purposes, it is highly desirable to know how different structured linearizations affect the accuracy of computed eigenelements. Thus the selection of an optimal or a near optimal structured linearization is an important step in the solution process of a structured polynomial eigenvalue problem. The sensitivity analysis of eigenvalues of structured matrix polynomials with respect to structure preserving perturbation has been investigated in [4]. It also provides a recipe for choosing structured linearizations whose eigenvalues are almost as sensitive to structure preserving perturbations as that of the structured matrix polynomials.

To complete the investigation, in this paper we analyze structured backward errors of approximate eigenelements of symmetric, skew-symmetric, Hermitian, skew-Hermitian, T-even, T-odd, H-even and H-odd polynomials. These structures are defined in Table 1. The main contribution of this paper is as follows.

First, we derive explicit computable expressions for the structured backward errors of approximate eigenelements of structured matrix polynomials. We also construct a minimal structured perturbation so that an approximate eigenelement is the exact eigenelement of the structured perturbed polynomial. These results generalize similar results in [3] obtained for structured matrix pencils.

Second, we analyze structured pseudospectra of structured matrix polynomials and establish a partial equality between structured and unstructured pseudospectra. Similar study for palindromic matrix polynomials has been carried out in [2], see also [1,8].

Third, we consider structured linearizations that preserve spectral symmetry of a structured matrix polynomial and compare the structured backward errors of approximate eigenelements with that of the structured polynomial. For example, a T-even matrix polynomial admits T-even as well as T-odd linearizations both of which preserve the spectral symmetry of the T-even polynomial. We show that structured linearizations that minimize the structured condition numbers of eigenvalues also minimize the structured backward errors of approximate eigenelements. We show that bad effect, if any, of a structure preserving linearization on the structured backward errors of approximate eigenelements can be neutralized by considering a complementary structured linearization. For example, when P is a T-even polynomial, we show that any T-even linearization is optimal for eigenvalues λ of P such that $|\lambda| \leqslant 1$, and any T-odd linearization is optimal for eigenvalues λ such that $|\lambda| \geqslant 1$. In such a case, we show that the backward error of an approximate eigenelement of the linearization differ from that of P by no more than a factor of 2. We show that similar results hold for other structured polynomials as well.