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On the state of behaviors

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Abstract

The theme of the present paper is the study of the concept of state and the corresponding state maps in the context of Willems' behavioral theory. We concentrate on Markovian system and their representation in terms of first order difference or differential systems. We follow by a full analysis of the special case of state systems, the embedding of a linear system in a state system via the use of state maps arriving at state representations or, equivalently, to a realization theory for behaviors. Minimality is defined and characterized and a state space isomorphism theorem is established. Realization procedures based on the shift realization are developed as well as a rigorous analysis of the construction of state maps. The paper owes much to Rapisarda and Willems [P. Rapisarda, J.C. Willems, State maps for linear systems, SIAM J. Contr. Optim. 35 (1997) 1053–1091].

Keywords: Behaviors; Behavior homomorphisms; Markovian systems; State systems; Realization theory; State maps; State space isomorphisms

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1. Introduction

The concept of state is a basic one in systems theory. Nontrivial systems have a memory of past events and the scope of this memory is crystallized in the concept of state. As a byproduct, using the introduced state variables, we are led to first order representations, i.e. realizations, of the system, a construction extremely useful in simulation, control and design applications. The usefulness of first order representations is such that in many cases they are taken as a starting point for the analysis of systems. However, alternative points of view have been adopted in the course of time, the most prominent of which are associated with the names of Kalman, Rosenbrock and Willems. It was Kalman who formalized the input/output approach and put realization theory as a cornerstone of the general linear systems theory. Rosenbrock realized that most linear systems are modelled in terms of higher order equations and, with the introduction of polynomial system matrices, he constructed a beautiful theory that did not take input/output considerations as basic. This was partly hidden because it was a result of noncontrollability or nonobservability which were expressed in terms of noncoprimeness of certain polynomial matrices. It was Willems, in a series of seminal papers who took the final step of disposing with inputs, outputs and 1st order representations, and focused on manifest, or external, variables. Latent, or auxiliary, variables were introduced in this framework in order to accommodate the many practical cases in which, in order to model the behavior of a system, auxiliary dynamics involving additional variables must also be used.

Of course the concept of state with all its usefulness had to be accommodated also in the behavioral setting. The concepts of state systems and state representations were introduced, in the behavioral setting, in Willems [26,27]. There one can find the characterizations of state systems as those having first order representations. This characterization was not constructive. To construct a first order representation for a linear dynamical system, state maps were introduced in Rapisarda and Willems [21].

The aim of this paper is to take another, somewhat different, look at the concepts of state systems, state representations and state maps. The approach taken here to the construction of state maps and 1st order representations, i.e. realizations, of behaviors is based on the theory of polynomial models and its application to behaviors. In particular, we shall employ the characterization of behavior homomorphisms and the analysis of their invertibility properties, as developed in Fuhrmann [10,11].

The paper is structured as follows. In Section 2, we shall collect some preliminary results about polynomial models, the shift realization, and reduction to dual Brunovsky form via output injection. For the analysis of state maps, we review the basic results on behavior homomorphisms and the role played by doubly unimodular embeddings in the analysis of their invertibility properties. Finally, we consider the class of state to output maps studied in Hautus and Heymann [15], and their connection to rational models and autonomous behaviors.

In Section 3 we characterize Markovian systems and their generalization, i.e. l-Markovian systems or equivalently l-memory span systems. We show that the analysis of l-Markovian systems can be reduced to the special case of autonomous system and then to the case of autonomous systems in dual Brunovsky form.

Section 4 is the core of the paper. We begin by analyzing the construction of state maps for autonomous dynamical systems. This analysis is the prototype for the general case. We proceed by showing how to construct state maps for an arbitrary behavior. This uses realizations of autonomous behaviors, doubly coprime factorizations and behavior homomorphisms. We conclude by showing how a special choice of basis, related to the dual Brunovsky form, leads to a simple