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On the analysis of a coupled kinetic-fluid model with local alignment forces

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Abstract

This paper studies global existence, hydrodynamic limit, and large-time behavior of weak solutions to a kinetic flocking model coupled to the incompressible Navier–Stokes equations. The model describes the motion of particles immersed in a Navier–Stokes fluid interacting through local alignment. We first prove the existence of weak solutions using energy and L^p estimates together with the velocity averaging lemma. We also rigorously establish a hydrodynamic limit corresponding to strong noise and local alignment. In this limit, the dynamics can be totally described by a coupled compressible Euler – incompressible Navier–Stokes system. The proof is via relative entropy techniques. Finally, we show a conditional result on the large-time behavior of classical solutions. Specifically, if the mass-density satisfies a uniform in time integrability estimate, then particles align with the fluid velocity exponentially fast without any further assumption on the viscosity of the fluid.

Keywords: Kinetic-fluid coupled equations; Asymptotic behavior; Flocking; Hydrodynamical limit

1. Introduction

In the animal kingdom, one can find several species where the action of individuals leads to large coherent structures and where there are no external forces or "leader" guiding the interaction. Perhaps the most famous examples are flocks of birds, schools of fish, or insect swarms. However, similar phenomena in self-organization are also relevant for bacteria, in robotic engineering, and in material science. The past decade has witnessed a massive growth in the attempts to develop mathematical models capturing these types of phenomena. These models are usually based on incorporating different mechanisms of interaction between the individuals such as local repulsion, long-range attraction, and alignment. These Individual Based Models lead to macroscopic descriptions by means of mean-field limit scalings, see [7] for a review. These continuum descriptions can be written as kinetic equations in which there is a mechanism of interaction in the velocity or orientation vector. A very simple idea implementing the consensus

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mechanism in velocity was introduced by Cucker and Smale in [9] and improved recently in [28]. These models take into account nonlocal interactions of the particles by averaging in velocity space. Here, we will focus on a much stronger local averaging of the velocity vector and the effect of a fluid in the tendency to consensus. We will explain the relation to these classical models of alignment below.

The model under consideration governs the motion of particles immersed in a Navier–Stokes fluid interacting through local alignment. By local alignment, we mean that each particle actively tries to align its velocity to that of its closest neighbors. The particles and fluid are coupled through linear friction. If we let $f = f(x, \xi, t)$ be the one-particle distribution function at a spatial periodic domain $x \in \mathbb{T}^3$, $\xi \in \mathbb{R}^3$ at time t, and $u = u(x, t)$ be the bulk velocity of fluid, then our model reads

$$\begin{aligned}
&\partial_t f + \xi \cdot \nabla_x f = \alpha \nabla_\xi \cdot \big[(\xi - u) f\big] + \beta \nabla_\xi \cdot \big[(\xi - u_f) f\big] + \sigma \Delta_\xi f \\
&\partial_t u + u \cdot \nabla_x u + \nabla_x p = \mu \Delta_x u - \alpha \rho_f (u - u_f) \\
&\nabla_x \cdot u = 0
\end{aligned} \tag{1.1}$$

subject to initial data

$$f(x, \xi, 0) = f_0(x, \xi), \qquad u(x, 0) = u_0(x), \tag{1.2}$$

where $\alpha, \beta, \sigma > 0$ are constants, and ρ_f and u_f denote the average local density and velocity, respectively

$$\rho_f := \int_{\mathbb{R}^3} f \, d\xi, \qquad \rho_f u_f := \int_{\mathbb{R}^3} \xi f \, d\xi. \tag{1.3}$$

The model (1.1) contains as particular cases two previously studied models in the literature. If $\beta = 0$, the model reduces to the fluid-particle model studied in [16,17], see also [3,8,18,26,27]. They analyzed the existence of weak solutions and their hydrodynamic limit. On the other hand, if $\alpha = 0$, (1.1) decouples and becomes the kinetic flocking model studied in [21–23]. This latter series of papers establishes existence of weak solutions and hydrodynamic limit, but leaves out the question of large-time behavior.

In this paper, we shall be concerned with the case α, $\beta > 0$. This introduces new difficulties compared to the previous studies, requiring non trivial arguments to overcome them. To prove existence of weak solutions to (1.1), the main challenges are posed by the product $f u_f$ and the lack of regularity on u. In the first case, weak compactness of $f u_f$ is not trivial as there does not seem to be any available regularity in a spatial domain. Moreover, u_f is only defined on regions with $\varrho_f > 0$ and hence does not belong to any L^p-space. In this paper, we will obtain the needed compactness from the velocity averaging lemma together with some technical arguments. This part of the proof will be similar to the existence proof in [21] for (1.1) with $\alpha = 0$. However, the coupling with the Navier–Stokes equations introduces new problems that are not straightforward to handle.

Since Eq. (1.1) is posed in $2d + 1$ dimensions, finding an approximate solution is computationally expensive. For this reason, it is of interest to identify regimes where the complexity of the equations reduces. In this paper, we shall rigorously identify one such regime corresponding to strong noise and local alignment. That is, the case where $\beta \sim \sigma \sim \varepsilon^{-1}$, where ε is a small number. We will establish that in this case f is close to a thermodynamical equilibrium $f \sim c_0 \varrho_f e^{-|u_f - \xi|^2/2}$ and that the dynamics can be well approximated by a compressible Euler equation for (ϱ_f, u_f) coupled to the incompressible Navier–Stokes equations for u (see Section 2.2 for clarity). We will achieve this result by establishing a relative entropy inequality. Though this type of inequality was originally devised in [10] to prove weak-strong uniqueness results, it has also been successfully applied to hydrodynamic limits for kinetic equations [14,24,29]. The perhaps most relevant study is [22], where (1.1) with $\alpha = 0$ is studied. However, with $\beta > 0$, deriving a relative entropy bound is more involved and requires completely new arguments that does not have a kin in the literature.

For the estimates of large-time behavior of solutions, when $\beta = 0$, i.e., no local alignment force, the particle-fluid equations (1.1) reduce to the Vlasov–Navier–Stokes–Fokker–Planck equations. For this system, classical solutions near Maxwellians converging asymptotically to them were constructed in [15]. More recently, the incompressible Euler–Fokker–Planck equations ($\beta = 0$ and $\mu = 0$) were treated in [5] showing the existence of a unique classical solution near Maxwellians converging to them. On the other hand, without the diffusive term ($\sigma = 0$), the particle-fluid system has no trivial equilibria, and as a consequence the previous arguments used in [5,15] for the estimates of large-time behavior can not be applied. The large-time behavior of the Vlasov–Navier–Stokes equations, to our knowledge,