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Pointwise bounds and blow-up for nonlinear polyharmonic inequalities

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Abstract

We obtain results for the following question where $m \geqslant 1$ and $n \geqslant 2$ are integers.

Question. For which continuous functions $f : [0, \infty) \to [0, \infty)$ does there exist a continuous function $\varphi : (0, 1) \to (0, \infty)$ such that every C^{2m} nonnegative solution $u(x)$ of

$$0 \leqslant -\Delta^m u \leqslant f(u) \quad \text{in } B_2(0) \setminus \{0\} \subset \mathbb{R}^n$$

satisfies

$$u(x) = O\big(\varphi\big(|x|\big)\big) \quad \text{as } x \to 0$$

and what is the optimal such φ when one exists?

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Résumé

Nous obtenons des résultats pour la question suivante, avec $m \geqslant 1$ et $n \geqslant 2$ entiers.

Question. Pour quelles fonctions continues $f : [0, \infty) \to [0, \infty)$ existe-t-il une fonction continue $\varphi : (0, 1) \to (0, \infty)$ telle que chaque solution C^{2m} non-negative $u(x)$ de

$$0 \leqslant -\Delta^m u \leqslant f(u) \quad \text{dans } B_2(0) \setminus \{0\} \subset \mathbb{R}^n$$

satisfasse à

$$u(x) = O\big(\varphi\big(|x|\big)\big) \quad \text{lorsque } x \to 0,$$

et quelle est la meilleure de ces fonctions φ quand elle existe ?

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1. Introduction

In this paper we consider the following question where $m \geqslant 1$ and $n \geqslant 2$ are integers.

Question 1. For which continuous functions $f : [0, \infty) \to [0, \infty)$ does there exist a continuous function $\varphi : (0, 1) \to (0, \infty)$ such that every C^{2m} nonnegative solution $u(x)$ of

$$0 \leqslant -\Delta^m u \leqslant f(u) \quad \text{in } B_2(0) \setminus \{0\} \subset \mathbb{R}^n \tag{1.1}$$

satisfies

$$u(x) = O\big(\varphi\big(|x|\big)\big) \quad \text{as } x \to 0 \tag{1.2}$$

and what is the optimal such φ when one exists?

We call a function φ with the above properties a pointwise a priori bound (as $x \to 0$) for C^{2m} nonnegative solutions $u(x)$ of (1.1).

As we shall see, when φ in Question 1 is optimal, the estimate (1.2) can sometimes be sharpened to

$$u(x) = o\big(\varphi\big(|x|\big)\big) \quad \text{as } x \to 0.$$

Remark 1.1. Let

$$\Gamma(r) = \begin{cases} r^{-(n-2)}, & \text{if } n \geqslant 3; \\ \log \frac{5}{r}, & \text{if } n = 2. \end{cases} \tag{1.3}$$

Since $u(x) = \Gamma(|x|)$ is a positive solution of $-\Delta^m u = 0$ in $B_2(0) \setminus \{0\}$, and hence a positive solution of (1.1), any pointwise a priori bound φ for C^{2m} nonnegative solutions $u(x)$ of (1.1) must be at least as large as Γ, and whenever $\varphi = \Gamma$ is such a bound it is necessarily an optimal bound.

Some of our results for Question 1 can be generalized to allow the function f in (1.1) to depend nontrivially on x and the partial derivatives of u up to order $2m - 1$. (See the second paragraph after Proposition 2.1.)

We also consider the following analog of Question 1 when the singularity is at ∞ instead of at the origin.

Question 2. For which continuous functions $f : [0, \infty) \to [0, \infty)$ does there exist a continuous function $\varphi : (1, \infty) \to (0, \infty)$ such that every C^{2m} nonnegative solution $v(y)$ of

$$0 \leqslant -\Delta^m v \leqslant f(v) \quad \text{in } \mathbb{R}^n \setminus B_{1/2}(0) \tag{1.4}$$

satisfies

$$v(y) = O\big(\varphi\big(|y|\big)\big) \quad \text{as } |y| \to \infty$$

and what is the optimal such φ when one exists?

The m-Kelvin transform of a function $u(x)$, $x \in \Omega \subset \mathbb{R}^n \setminus \{0\}$, is defined by

$$v(y) = |x|^{n-2m} u(x) \quad \text{where } x = y/|y|^2. \tag{1.5}$$

By direct computation, $v(y)$ satisfies

$$\Delta^m v(y) = |x|^{n+2m} \Delta^m u(x). \tag{1.6}$$

See [17, p. 221] or [18, p. 660]. Using this fact and some of our results for Question 1, we will obtain results for Question 2.

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