Available online at www.sciencedirect.com

ScienceDirect

Ann. I. H. Poincaré – AN 32 (2015) 1099–1123

ANNALES DE L'INSTITUT HENRI POINCARÉ
ANALYSE NON LINÉAIRE

www.elsevier.com/locate/anihpc

The two-species Vlasov–Maxwell–Landau system in $\mathbb{R}^3$ ☆

Yanjin Wang

School of Mathematical Sciences, Xiamen University, Xiamen, Fujian 361005, China

Received 21 December 2013; received in revised form 14 May 2014; accepted 22 May 2014

Available online 11 June 2014

Abstract

We consider the global classical solutions near the Maxwellians to the two-species Vlasov–Maxwell–Landau system in the whole space. It is shown that the cancelation properties between two species coupled with the electric effect yield the faster time decay of the electric field, which leads to our construction of global solutions.

MSC: 82C40; 82D05; 82D10; 35B40

Keywords: Vlasov–Maxwell–Landau system; Global solution; Time decay; Energy method

1. Introduction

The dynamics of charged dilute particles (e.g., electrons and ions) is described by the Vlasov–Maxwell–Landau system:

$$\begin{aligned}
&\partial_t F_+ + v\cdot\nabla_x F_+ + (E + v\times B)\cdot\nabla_v F_+ = Q(F_+,F_+) + Q(F_-,F_+),\\
&\partial_t F_- + v\cdot\nabla_x F_- - (E + v\times B)\cdot\nabla_v F_- = Q(F_+,F_-) + Q(F_-,F_-),\\
&F_\pm(0,x,v) = F_{0,\pm}(x,v).
\end{aligned}\tag{1.1}$$

Here $F_\pm(t,x,v)\ge 0$ are the number density functions for the ions $(+)$ and electrons $(-)$ respectively, at time $t\ge 0$, position $x=(x_1,x_2,x_3)\in\mathbb{R}^3$ and velocity $v=(v_1,v_2,v_3)\in\mathbb{R}^3$. The collision between charged particles is given by the Landau (Fokker–Planck) operator:

$$Q(G_1,G_2)(v) = \nabla_v\cdot\int_{\mathbb{R}^3}\Phi\big(v-v'\big)\big(G_1\big(v'\big)\nabla_v G_2(v) - G_2(v)\nabla_{v'}G_1\big(v'\big)\big)dv',\tag{1.2}$$

☆ This research was supported by the National Natural Science Foundation of China (No. 11201389), the Natural Science Foundation of Fujian Province of China (No. 2012J05011), the Specialized Research Fund for the Doctoral program of Higher Education (No. 20120121120023), and the Fundamental Research Funds for the Central Universities (No. 2013121002).

E-mail address: yanjin_wang@xmu.edu.cn.

http://dx.doi.org/10.1016/j.anihpc.2014.05.005

0294-1449/

where

$$\Phi(v)=\frac{1}{|v|}\left(I-\frac{v\otimes v}{|v|^2}\right). \tag{1.3}$$

The self-consistent electromagnetic field $(E(t,x),B(t,x))$ in (1.1) is coupled with $F_\pm(t,x,v)$ through the Maxwell system

$$\begin{aligned}
&\partial_t E-\nabla_x\times B=-\int_{\mathbb{R}^3} v(F_+-F_-)\,dv,\quad \nabla_x\cdot E=\int_{\mathbb{R}^3}(F_+-F_-)\,dv,\\
&\partial_t B+\nabla_x\times E=0,\quad \nabla_x\cdot B=0,\\
&E(0,x)=E_0(x),\qquad B(0,x)=B_0(x).
\end{aligned} \tag{1.4}$$

It turns out that all the physical constants will not create essential mathematical difficulties along our analysis, for notational simplicity, we have normalized all constants in the Vlasov–Maxwell–Landau system to be one. Accordingly, we normalize the global Maxwellian as

$$\mu(v)\equiv\mu_+(v)=\mu_-(v)=\mathrm{e}^{-|v|^2}. \tag{1.5}$$

We define the standard perturbation $f_\pm(t,x,v)$ to μ as

$$F_\pm=\mu+\sqrt{\mu}f_\pm. \tag{1.6}$$

Letting $f(t,x,v)=\binom{f_+(t,x,v)}{f_-(t,x,v)}$, the Vlasov–Maxwell–Landau system for the perturbation now takes the form

$$\begin{aligned}
&\{\partial_t+v\cdot\nabla_x+q_0(E+v\times B)\cdot\nabla_v\}f-2E\cdot v\sqrt{\mu}q_1+Lf=\Gamma(f,f)+q_0E\cdot vf,\\
&\partial_t E-\nabla_x\times B=-\int_{\mathbb{R}^3} v\sqrt{\mu}(f_+-f_-)\,dv,\quad \nabla_x\cdot E=\int_{\mathbb{R}^3}\sqrt{\mu}(f_+-f_-)\,dv,\\
&\partial_t B+\nabla_x\times E=0,\quad \nabla_x\cdot B=0,
\end{aligned} \tag{1.7}$$

for the matrix $q_0=\mathrm{diag}(1,-1)$ and the vector $q_1=\binom{1}{-1}$. For $g=\binom{g_1}{g_2}$, the linearized collision operator Lg in (1.7) is given by the vector

$$Lg\equiv\begin{pmatrix}L_+g\\L_-g\end{pmatrix}\equiv-\begin{pmatrix}\frac{2}{\sqrt{\mu}}Q(\mu,\sqrt{\mu}g_1)+\frac{1}{\sqrt{\mu}}Q(\sqrt{\mu}(g_1+g_2),\mu)\\ \frac{2}{\sqrt{\mu}}Q(\mu,\sqrt{\mu}g_2)+\frac{1}{\sqrt{\mu}}Q(\sqrt{\mu}(g_1+g_2),\mu)\end{pmatrix}. \tag{1.8}$$

For $g=\binom{g_1}{g_2}$ and $h=\binom{h_1}{h_2}$, the nonlinear collision operator $\Gamma(g,h)$ in (1.7) is given by the vector

$$\Gamma(g,h)\equiv\begin{pmatrix}\Gamma_+(g,h)\\ \Gamma_-(g,h)\end{pmatrix}\equiv\begin{pmatrix}\frac{1}{\sqrt{\mu}}Q(\sqrt{\mu}(g_1+g_2),\sqrt{\mu}h_1)\\ \frac{1}{\sqrt{\mu}}Q(\sqrt{\mu}(g_1+g_2),\sqrt{\mu}h_2)\end{pmatrix}. \tag{1.9}$$

For the Landau operator (1.2), we define

$$\sigma^{ij}(v)=\Phi^{ij}*\mu=\int_{\mathbb{R}^3}\Phi^{ij}(v-v')\mu(v')\,dv'. \tag{1.10}$$

We denote $|\cdot|_2$ to be the $L^2(\mathbb{R}^3_v)$ norm, and we define $L^2_\sigma(\mathbb{R}^3_v)$ to be the space with norm

$$|f|_\sigma^2=\int_{\mathbb{R}^3}\left[\sigma^{ij}\partial_i f\partial_j f+\sigma^{ij}v_iv_jf^2\right]dv. \tag{1.11}$$

From Lemma 3 in [9], we have

$$C^{-1}|f|_\sigma\le\left|\langle v\rangle^{-\frac12}f\right|_2+\left|\langle v\rangle^{-\frac32}\nabla_v f\cdot\frac{v}{|v|}\right|_2+\left|\langle v\rangle^{-\frac12}\nabla_v f\times\frac{v}{|v|}\right|_2\le C|f|_\sigma \tag{1.12}$$