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Image recovery via geometrically structured approximation

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ABSTRACT

In recent years, the ℓ_1 norm based regularization has been one promising technique for solving many ill-posed inverse problems in image recovery. As the performance gain of these methods over linear methods comes from the separate process for smooth image regions and image discontinuities, their performance largely depends on how accurate such separation is. However, there is a lot of ambiguities between smooth image regions and image discontinuities when only degraded images are available. This paper aims at developing new wavelet frame based image regularization to resolve such ambiguities by exploiting geometrical regularities of image discontinuities. Based on the geometrical connectivity constraint on image discontinuities, an alternating iteration scheme is proposed which is simple in implementation and efficient in computation. The experiments show that the results from the proposed regularization method are compared favorably against that from several existing ℓ_1 norm or ℓ_0 norm based image regularizations.

1. Introduction

Image recovery is one fundamental process with a wide range of applications. Image recovery aims at recovering an image of high-quality from its degraded measurement with noise. The degraded measurement $\mathbf{g}$ is often modeled by applying a linear operator $\mathbf{A}$ to the original signal $\mathbf{f}$:

$$\mathbf{g} = \mathbf{A}\mathbf{f} + \boldsymbol{\epsilon}, \tag{1}$$

where $\boldsymbol{\epsilon}$ denotes the measurement noise which is typically assumed to be white Gaussian noise. In most image restoration tasks, the linear operator $\mathbf{A}$ is either ill-posed or non-invertible thus a straightforward inversing process will either significantly amplify the noise or generating artifacts in the solution. For example, for image deblurring, $\mathbf{A}$ is a convolution operator by a low-pass filter that will significantly attenuate/erase high-frequency components of an image. For image inpainting, $\mathbf{A}$ is a projection operator that maps the

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full set of image pixels to its subset. To solve these ill-posed image restoration problems, certain prior information of the image for recovery is needed to regularize the recovery process.

By assuming that images can be well approximated by smooth functions, traditional linear methods, e.g. Tikhonov regularizations, find the solution of (1) via solving a squared ℓ_2 norm based minimization:

$$\min_{\mathbf{f}} \|\mathbf{Af}-\mathbf{g}\|_2^2 + \lambda\|\mathbf{\Gamma f}\|_2^2, \tag{2}$$

where $\mathbf{\Gamma}$ is an operator such that the coefficients $\mathbf{\Gamma f}$ are close to zero. One often used operator is the difference operator. However, most images of interest cannot be well approximated by smooth functions, as the content of an image is actually dominated by image edges which show rapid intensity changes in its neighborhood. Due to omitting image discontinuities, image edges in the solution obtained from (2) are usually smoothed out.

Suppose the locations of image discontinuities are known with oracles, then the image $\mathbf{f}$ can be recovered with sharp image edges by solving the following optimization problem:

$$\min_{\mathbf{f}} \|\mathbf{Af}-\mathbf{g}\|_2^2 + \lambda\|(\mathbf{\Gamma f})_{\Omega^c}\|_2^2 \tag{3}$$

where Ω is the index set of image pixels on image discontinuities, and Ω^c denotes its complement. Thus, as long as there are sufficient entries in Ω^c, the solution from (3) is a well-posed one, and image discontinuities are well kept in the solution. Clearly, the critical part in (3) is then how to accurately estimate the index set Ω, or equivalently, how to accurately detect the locations of image discontinuities. For image recovery, such a task is a difficult one as only degraded images are available.

1.1. Treatment of image discontinuities in existing methods

The concept of (3) is indeed implicitly utilized in the ℓ_1 norm based regularizations, which generally yield better results than the Tikhonov regularization (2). The most often used ℓ_1 norm based regularization to find the solution of (1) is by solving the following optimization problem:

$$\frac{1}{2}\|\mathbf{Af}-\mathbf{g}\|_2^2 + \lambda\|\mathbf{\Gamma f}\|_1, \tag{4}$$

where $\mathbf{\Gamma}$ is some operator such that most entries of $\mathbf{\Gamma f}$ are zeros or close to zero. The most often seen operators include difference operators with different orders and the analysis operators of wavelet frames. It can be seen that when images are well approximated by piece-wise smooth functions, the values of $\mathbf{\Gamma f}$ on smooth image regions will be zero or close to zero and the values of $\mathbf{\Gamma f}$ on image discontinuities will have large magnitude. As ℓ_1 norm can be used as a sparsity-prompting function [9,21], the model (4) seeks a solution that have a small percentage of entries with large magnitude and expects the pixels corresponding to these entries are exactly on image discontinuities. However, for smooth image regions, the penalty of ℓ_1 norm is often not large enough to avoid those estimations with some artificial discontinuities. As a result, the model (4) sometimes introduces un-wanted artifacts, e.g. artificial image edges in smooth image regions.

The concept of (3) is explicitly exploited in the optimization model proposed in [2] for recovering images with piece-wise smoothness:

$$\min_{\Omega,\mathbf{f}} \frac{1}{2}\|\mathbf{Af}-\mathbf{g}\|_2^2 + \lambda\|(\mathbf{Wf})_{\Omega^c}\|_2^2 + \mu\|(\mathbf{Wf})_{\Omega}\|_1, \tag{5}$$

where $\mathbf{W}$ denotes the analysis operator of a wavelet tight frame in high-pass channels, and Ω denotes the index set of the entries of $\mathbf{Wf}$ with significant magnitude. As image discontinuities can be well modeled by wavelet frame coefficients with large magnitude, the model (5) uses the square of ℓ_2 norm to regularize