Appl. Comput. Harmon. Anal. 41 (2016) 384–411

Contents lists available at ScienceDirect

Applied and Computational Harmonic Analysis

www.elsevier.com/locate/acha

Adaptive local iterative filtering for signal decomposition and instantaneous frequency analysis

Antonio Cicone [a,*], Jingfang Liu [b], Haomin Zhou [b]

[a] *Marie Curie fellow of the Istituto Nazionale di Alta Matematica, DISIM, Università degli Studi dell'Aquila, via Vetoio n. 1, 67100 L'Aquila, Italy*
[b] *School of Mathematics, Georgia Institute of Technology, 686 Cherry St. Atlanta, GA 30332, USA*

A R T I C L E   I N F O

Article history:
Received 2 April 2015
Received in revised form 5 October 2015
Accepted 1 March 2016
Available online 7 March 2016
Communicated by the Spec. Issue Guest Editor

Keywords:
Iterative Filtering
Empirical mode decomposition
Fokker–Planck equations
Instantaneous frequency

A B S T R A C T

Time–frequency analysis for non-linear and non-stationary signals is extraordinarily challenging. To capture features in these signals, it is necessary for the analysis methods to be local, adaptive and stable. In recent years, decomposition based analysis methods, such as the empirical mode decomposition (EMD) technique pioneered by Huang et al., were developed by different research groups. These methods decompose a signal into a finite number of components on which the time–frequency analysis can be applied more effectively.

In this paper we consider the Iterative Filtering (IF) approach as an alternative to EMD. We provide sufficient conditions on the filters that ensure the convergence of IF applied to any L^2 signal. Then we propose a new technique, the Adaptive Local Iterative Filtering (ALIF) method, which uses the IF strategy together with an adaptive and data driven filter length selection to achieve the decomposition. Furthermore we design smooth filters with compact support from solutions of Fokker–Planck equations (FP filters) that can be used within both IF and ALIF methods. These filters fulfill the derived sufficient conditions for the convergence of the IF algorithm. Numerical examples are given to demonstrate the performance and stability of IF and ALIF techniques with FP filters. In addition, in order to have a complete and truly local analysis toolbox for non-linear and non-stationary signals, we propose new definitions for the instantaneous frequency and phase which depend exclusively on local properties of a signal.

1. Introduction

Data and signal analysis has become increasingly important these days. Decomposing signals and finding features of data is quite challenging especially when the data is non-stationary and it is generated by a non-linear system. Time–frequency analysis has been substantially studied in the past, we refer to [3]

* Corresponding author.
E-mail addresses: antonio.cicone@univaq.it (A. Cicone), jingfang.liu1@gmail.com (J. Liu), hmzhou@math.gatech.edu (H. Zhou).

http://dx.doi.org/10.1016/j.acha.2016.03.001

and [11] for more information on this rich subject. Traditionally, Fourier spectral analysis as well as wavelet transforms have been commonly used. Both approaches are effective and easy to implement, however there are some limitations. Fourier transform works well when the data is periodic or stationary and the associated systems are linear, it cannot deal with non-stationary signals or data from non-linear systems. The wavelet transform is also a linear analysis tool. Both approaches use predetermined bases and they are not designed to be data-adaptive. Hence these techniques often cannot achieve desirable results for non-linear and non-stationary signals.

In the last decade, several decomposition techniques have been proposed to analyze non-linear and non-stationary signals. All these methods share the same approach: first they decompose a signal into simpler components and then they apply a time–frequency analysis to each component separately. The signal decomposition can be achieved in two ways: by iteration or by optimization.

The first iterative algorithm of this kind, the empirical mode decomposition (EMD), was introduced by Huang et al. [19] in 1998. This method aims to iteratively decompose a signal into a finite sequence of intrinsic mode functions (IMFs) whose instantaneous frequencies are well behaved. We will come back to instantaneous frequency later in this paper, let us instead describe the iterative structure of EMD which is called the Sifting Process.

Let $\mathcal{L}$ be an operator getting the moving average of a signal $f(x)$ and $\mathcal{S}$ be an operator capturing the fluctuation part $\mathcal{S}(f)(x) = f(x) - \mathcal{L}(f)(x)$. Then the first IMF produced by the sifting process is

$$I_1(x) = \lim_{n\to\infty} \mathcal{S}_{1,n}\left(f_n\right)(x) \tag{1}$$

where $f_n(x) = \mathcal{S}_{1,n-1}(f_{n-1})(x)$ and $f_1(x) = f(x)$. Here the limit is reached so that applying $\mathcal{S}$ one more time does not change the signal.

The subsequent IMFs are obtained one after another by

$$I_k(x) = \lim_{n\to\infty} \mathcal{S}_{k,n}(r_n)(x) \tag{2}$$

where $r_n(x) = \mathcal{S}_{k,n-1}(r_{n-1})(x)$ and $r_1(x) = r(x)$ which is the remainder $f(x) - I_1(x) - \ldots - I_{k-1}(x)$. The sifting process stops when $r(x) = f(x) - I_1(x) - I_2(x) - \ldots - I_m(x)$ becomes a trend signal, which means it has at most one local maximum or minimum. So the decomposition of $f(x)$ is

$$f(x) = \sum_{j=1}^{m} I_j(x) + r(x) \tag{3}$$

In this iterative process the moving average $\mathcal{L}(f)(x)$ is given by the mean function of the upper envelope and the lower envelope, which are given by cubic splines connecting local maxima and local minima of $f(x)$ respectively. However, this method is not stable under perturbations since cubic splines are used repeatedly in the iteration. To overcome this issue, Huang et al. developed the Ensemble Empirical Mode Decomposition (EEMD) [38] where the IMFs are taken as the mean of many different trials. In each trial a random perturbation is artificially added to the original signal. More details on EMD method and its analysis can be found, for instance, in [16] and [17,27,6,29,25,26,7].

Another iterative decomposition technique is the Iterative Filtering (IF) method which is inspired by EMD [21]. It uses the same algorithm framework as the original EMD, but the moving average of a signal $f(x), x \in \mathbb{R}$, is derived by the convolution of $f(x)$ with low pass filters, for example the double average filter $a(t)$ given by

$$a(t) = \frac{l+1-|t|}{(l+1)^2}, \quad t \in [-l, l] \tag{4}$$