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Letter to the Editor

## Stable recovery of analysis based approaches $\stackrel{\diamond}{\sim}$

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ABSTRACT

## A R T I C L E I N F O

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## 1. Introduction

In compressed sensing, a vector  $x \in \mathbb{R}^n$  is called *k-sparse* if the number of its nonzero entries is at most  $k \ll n$ . If a vector can be approximated well by sparse vectors, it is called a *compressible* vector. Suppose that the measurements of a sparse vector x are given by

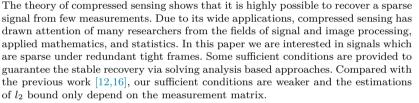
$$\boldsymbol{y} = A\boldsymbol{x} + \boldsymbol{w},$$

where  $\boldsymbol{w} \in \mathbb{R}^m$  is the noise and A is an  $m \times n$  measurement matrix with  $m \leq n$ . The sparse vector  $\boldsymbol{x}$  can be recovered via solving convex minimization problems, provided that the measurement matrix A satisfies some conditions such as mutual incoherence property [9] or restricted isometry property [5].

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This paper mainly considers the case that x is not sparse itself but is compressible with respect to some given tight frame D. If the  $l_2$  norm of the noise w is bounded by  $\varepsilon$ , then the recovery problem can be formulated as the constrained analysis based approach:

$$\min_{\tilde{\boldsymbol{x}}\in\mathbb{R}^n} \|D\tilde{\boldsymbol{x}}\|_1 \quad \text{subject to} \quad \|A\tilde{\boldsymbol{x}}-\boldsymbol{y}\|_2 \leqslant \varepsilon.$$
(1.1)

The constrained analysis based approach is said to produce a *stable* estimation  $\hat{x}$  to the true vector x if

$$\|\hat{\boldsymbol{x}} - \boldsymbol{x}\|_2 \leqslant C_0 \left(\varepsilon + \frac{\|D\boldsymbol{x} - (D\boldsymbol{x})_k\|_1}{\sqrt{k}}\right),\tag{1.2}$$

where  $(D\boldsymbol{x})_k$  denotes the vector consisting of the largest k coefficients of  $D\boldsymbol{x}$  in magnitude and  $C_0$  is a constant. As a special case, if the coefficient vector  $D\boldsymbol{x}$  is at most k-sparse and there is no noise in the measurements, then  $\boldsymbol{x}$  is exactly recovered. A widely used condition that guarantees the stable signal recovery is the restricted isometry property adapted to D [3]. Let  $D^*$  be the complex conjugate of the transpose of a  $d \times n$  matrix D. Since D is often real-valued,  $D^*$  in this paper is simply the transpose of D. Then D is a tight frame for  $\mathbb{R}^n$  if and only if  $D^*D = I_n$ . The measurement matrix A satisfies the *restricted isometry property adapted to* D (D-RIP) if there is a constant  $0 < \delta_k < 1$  such that

$$(1 - \delta_k) \|\boldsymbol{v}\|_2^2 \leqslant \|A\boldsymbol{v}\|_2^2 \leqslant (1 + \delta_k) \|\boldsymbol{v}\|_2^2$$
(1.3)

holds for all  $\boldsymbol{v} \in \Sigma_k := \{D^*\boldsymbol{c} : \boldsymbol{c} \in \mathbb{R}^d, \|\boldsymbol{c}\|_0 \leq k\}$ . The smallest constant  $\delta_k$  satisfying (1.3) is called the D-RIP constant. When D is the identity matrix, the constant  $\delta_k$  is called the *restricted isometry property* (RIP) constant [5]. Therefore, the D-RIP is a natural extension of the RIP. Almost all random matrices, such as Gaussian, subgaussian, Bernoulli random matrices and subsampled Fourier matrices, satisfying the RIP also satisfy the D-RIP, up to multiplication by a random sign matrix (in the case of the random DFT, for example) [3]. It was first proved in [3] that the condition  $\delta_{2k} < 0.08$  guarantees the stable recovery of the  $l_1$  minimization model in (1.1) and the constant  $C_0$  in (1.2) depends on the D-RIP constant  $\delta_{2k}$ . This sufficient condition for stable recovery was later improved in [13,14].

Instead of solving (1.1) directly, many algorithms were proposed to solve the following unconstrained analysis based approach:

$$\min_{\tilde{\boldsymbol{x}}\in\mathbb{R}^n}\lambda\|D\tilde{\boldsymbol{x}}\|_1 + \frac{1}{2}\|A\tilde{\boldsymbol{x}} - \boldsymbol{y}\|_2^2.$$
(1.4)

See [10] for a recent overview on the analysis based approaches (1.4). Under some assumptions on the matrix A and the tight frame D, the relation between (1.1) and (1.4) has been discussed in [18]. Generally, the minimization of (1.1) and the minimization of (1.4) are not exactly the same. To the best of our knowledge, using D-RIP to guarantee the stable recovery, the condition on the D-RIP constant  $\delta_{2k}$  for solving (1.4) is stronger than that for solving (1.1). Under the condition that  $\|DA^*w\|_{\infty} \leq \frac{\lambda}{2}$  and  $\delta_{2k} < 0.0833$ , the minimization solution  $\hat{x}$  to (1.4) satisfies

$$\|\hat{\boldsymbol{x}} - \boldsymbol{x}\|_2 \leqslant C_1 \sqrt{k}\lambda + C_2 \frac{\|D\boldsymbol{x} - (D\boldsymbol{x})_k\|_1}{\sqrt{k}},\tag{1.5}$$

where the constant  $C_2$  depends on the D-RIP constant while the constant  $C_1$  depends on both the D-RIP constant and  $\|DD^*\|_{1,1}$  [12]. Here  $\|DD^*\|_{1,1}$  stands for the operator norm of  $DD^*$  acting on  $l_1$ , that is,

$$\|DD^*\|_{1,1} := \sup\{\|DD^*\boldsymbol{c}\|_1 : \boldsymbol{c} \in \mathbb{R}^d, \|\boldsymbol{c}\|_1 \leqslant 1\}.$$

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