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Multichannel deconvolution with long range dependence: Upper bounds on the L^p-risk $(1 \leq p < \infty)$

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A B S T R A C T

We consider multichannel deconvolution in a periodic setting with long-memory errors under three different scenarios for the convolution operators, i.e., super-smooth, regular-smooth and box-car convolutions. We investigate global performances of linear and hard-thresholded non-linear wavelet estimators for functions over a wide range of Besov spaces and for a variety of loss functions defining the risk. In particular, we obtain upper bounds on convergence rates using the L^p-risk $(1 \leq p < \infty)$. Contrary to the case where the errors follow independent Brownian motions, it is demonstrated that multichannel deconvolution with errors that follow independent fractional Brownian motions with different Hurst parameters results in a much more involved situation. An extensive finite-sample numerical study is performed to supplement the theoretical findings.

1. Introduction

We study multichannel deconvolution with errors following independent fractional Brownian motions (fBms). More specifically, consider the problem of recovering $f(\cdot) \in L^2(T)$, $T = [0, 1]$, on the basis of observing the following noisy convolutions, with known blurring functions $g_\ell(\cdot)$,

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$$dY_\ell(t) = K_\ell f(t)dt + \frac{\sigma_\ell}{n^{\alpha_\ell/2}} dB_{H_\ell}(t), \quad t \in T,\ \ell = 1, 2, \dots, M, \tag{1}$$

where σ_ℓ are known positive constants and the convolution operators K_ℓ are defined as

$$K_\ell f(t) := f * g_\ell(t) = \int_T g_\ell(t - x) f(x) dx, \quad t \in T,\ \ell = 1, 2, \dots, M. \tag{2}$$

Here, $B_{H_\ell}(\cdot)$ are independent standard fBms with *Hurst* parameters $H_\ell = 1 - \alpha_\ell/2 \in [1/2, 1)$, $\ell = 1, 2, \dots, M$; that is, for each $\ell = 1, 2, \dots, M$; $B_{H_\ell}(\cdot)$ is a Gaussian process with zero mean and covariance function

$$\mathbb{E}\big(B_{H_\ell}(s) B_{H_\ell}(t)\big) = \frac{1}{2}\big(|s|^{2H_\ell} + |t|^{2H_\ell} - |t - s|^{2H_\ell}\big), \quad s, t \in T,\ \ell = 1, 2, \dots, M.$$

The case where $M = 1$ corresponds to the fractional Gaussian noise model that can also be viewed as an approximation to the nonparametric regression model with long-range dependence (LRD) (cf. [1,2]). On the other hand, the case $H_\ell = 1/2$, $\ell = 1, \dots, M$; becomes the *multichannel* deconvolution with independent standard Brownian motion errors. This model has received attention in studies by [3–5] and [6].

We consider the following scenarios for the convolution operators K_ℓ, $\ell = 1, 2, \dots, M$; given by (2) in the Fourier domain where $\widetilde{f}(m) := \int_{\mathbb{R}} e^{-2\pi i m x} f(x)\, dx$.

1. *Smooth* convolutions such that, in the Fourier domain,

$$\big|\widetilde{K_\ell f}(m)\big| \asymp |m|^{-\nu_\ell} \exp\big\{-\theta_\ell |m|^{\beta_\ell}\big\} \big|\widetilde{f}(m)\big|, \tag{3}$$

   where $m \in \mathbb{R}$, $\ell = 1, 2, \dots, M$; $\beta_\ell > 0$ and $\theta_\ell \geq 0$. In particular, $\nu_\ell \in \mathbb{R}$ if $\theta_\ell > 0$ and $\nu_\ell > 0$ if $\theta_\ell = 0$. The key parameter is θ_ℓ, controlling the severity of the decay. The so-called super-smooth deconvolution or exponential decay occurs when $\theta_\ell > 0$ and the regular-smooth or polynomial case occurs when $\theta_\ell = 0$. In the regular-smooth case, each $\nu_\ell > 0$ corresponds to the so-called *degree of ill-posedness* (DIP) index with $\nu_\ell = 0$ representing the *direct* (or *well-posed*) case.
2. *Box-car* convolutions such that, in the Fourier domain,

$$\big|\widetilde{K_\ell f}(m)\big| = \frac{\sin(\pi m c_\ell)}{\pi m c_\ell} \big|\widetilde{f}(m)\big|, \quad m \in \mathbb{R},\ \ell = 1, 2, \dots, M; \tag{4}$$

   where $c_\ell > 0$ for each $\ell = 1, 2, \dots, M$.

Deconvolution is a common problem in many areas of signal and image processing which include, for instance, light detection and ranging (LIDAR) remote sensing and reconstruction of blurred images. LIDAR is a laser device which emits pulses, reflections of which are gathered by a telescope aligned with the laser. The return signal is used to determine the distance and the position of the reflecting material. However, if the system response function of the LIDAR is longer than the time resolution interval, then the measured LIDAR signal is blurred and the effective accuracy of the LIDAR decreases. This loss of precision can be corrected by deconvolution. In practice, measured LIDAR signals are corrupted by additional noise which renders direct deconvolution impossible. Moreover, if $M \geq 2$ (finite) LIDAR devices are used to recover a signal, then we talk about a *multichannel* deconvolution problem. The case where $M \geq 2$ in (1)–(2) and $H_\ell = 1/2$, $\ell = 1, \dots, M$; i.e., the problem of considering systems of convolution equations with independent errors, was first considered by [7] in order to evade the ill-posedness of the standard deconvolution model.

In the standard Brownian motion error case, a statistical use of the above idea was investigated by [8,3] who proposed adaptive wavelet thresholding estimators. In particular, if K_ℓ are regular-smooth convolutions,