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Letter to the Editor

Texture separation via a reference set

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ABSTRACT

Patch-based de-noising algorithms and patch manifold smoothing have emerged as efficient de-noising methods. This paper provides a new insight on these methods. We show how to extend them to separate oscillatory patterns that could be entangled. A collection of particular patches, that we call reference set, is selected by the user. We define a notion of similarity relative to this reference set that is used to extend the Non-Local Means (see Buades et al., 2005) [1] and the graph-based de-noising method (see Szlam et al., 2008) [12].

1. Introduction

The celebrated Non-Local Means (NL-Means) filter of Buades et al. [1] has proven to be a very efficient patch-based de-noising algorithm. The fundamental idea underneath it is to consider a natural image as a redundant collection of sub-images, also called patches. Each noisy patch is approximated by a weighted average of patches where the weights reflect the similarity between any two patches. The averaging defines a nonlocal operator that reduces the noise. Following the footsteps of Buades et al. [1], Gilboa et al. [4,5] have proposed nonlocal operators that are used in variational methods to decompose an image into a sum of a geometric component and a texture component. For instance, they propose a nonlocal version of the so-called Rudin–Osher–Fatemi model [14] that relies on a nonlocal extension of the total variation norm. Graph-cut techniques [3] are used to obtain the decomposition; the use of spectral graph theory for image segmentation was suggested by Shi and Malik [15]. Tools from differential geometry and analysis on manifolds have proven useful to understand the NL-Means filter as a special case within the diffusion geometry framework [13]. The grayscale-valued image is lifted and embedded in a higher-dimensional space by considering the manifold of patches. The grayscale value is interpreted as a function defined on the manifold. Although a patch is high-dimensional (e.g. a patch of size 7×7 is of dimension 49), in practice the intrinsic dimension is much smaller. The nonlocal weighted averaging of patches is understood as a smoothing on the manifold. Specifically, in [12] the heat equation on the manifold is used for de-noising.

In this work, our primary goal is not de-noising but separating oscillatory patterns. More precisely, we are interested in decomposing an image into two components; one similar to a user-specified pattern and a second one corresponding to a residual term that is free of this pattern. Our goal is to extract a user-specified oscillatory pattern whereas classic "object + texture" models [14,5] aim at separating geometric objects from textures in general without any distinction between oscillatory patterns. Furthermore, we are interested in resolving the intersection of two manifolds. Specifically, a patch extracted inside the intersection of the manifolds is a mixture of different patterns. We want to extract the component that is similar

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to the user-specified pattern. In that framework, our task is filtering according to the pattern of interest (it could be several patterns). The user-specified pattern is interpreted as a subset of the collection of all fixed-size patches extracted from the image. From now on, we refer to it as the "reference set". We propose to extend the NL-Means filter and its graph-based diffusion variants by taking into account the reference set. Any patch that is similar to the pattern of interest should not be "too far" from the reference set with respect to some patch-based metric. NL-Means uses the Euclidean metric between patches. In the graph-based method described in [12], a pixel is mapped to a feature vector whose coordinates are the responses of a filter bank with the patch centered at that pixel. Feature vectors are then compared using the Euclidean norm. In both methods, the affinity between pixels is a nonnegative decreasing function of the metric (e.g. a Gaussian function). NL-Means averages the value at each pixel by weighting the sum by the affinities. First, we propose to keep those weights but restrict the averaging over pixels whose patches are in the reference set. In addition to de-noising the image we are filtering oscillatory patterns according to the reference set. This defines an operator that we name "reference-based nonlocal filter operator". We show the limitations of this operator and propose a variant that we name "projected nonlocal filter operator". Second, we introduce a graph-based diffusion approach relative to the reference set. The pixels form the nodes of a graph. The edges are defined once we define a notion of affinity or neighborhood between nodes. This is defined relative to the reference set. This definition is crucial for computing efficiently the eigenfunctions of the Laplace–Beltrami operator of the whole set. We show that there exists an extension formula between the eigenfunctions on the reference set and the eigenfunctions on the whole set.

This paper is organized as follows. In Section 2, we present the reference-based nonlocal filter and the projected nonlocal filter operators and motivate our preference for the latter. In Section 3, the general framework of our graph-based diffusion approach relative to the reference set is explained. Details of the extension formula and its use for projection are given. In Section 4, we demonstrate our methods in separating oscillatory patterns and show that the eigenfunctions of the Laplace–Beltrami operator yield an embedding of the manifold of images into a low-dimensional meaningful space [7].

2. Reference-based nonlocal filter

In this section we first briefly review the NL-Means algorithm and then propose two modified versions that aim at separating possibly entangled oscillatory patterns. First, we extend the NL-Means model in such a way that we only compare patches to the reference set. This defines a nonlocal operator that we call reference-based nonlocal filter operator. This method performs decently when the desired texture is not mixed with other textures. We illustrate this point in Section 4. We then show how to overcome the limitations and propose another algorithm that we name projected nonlocal filter operator.

The NL-Means algorithm has been introduced in [1] for de-noising purposes. Any pixel, say $i \in I$, where I represents the set of all pixels inside the image, is associated to a fixed-size rectangle centered at it that we call $patch(i)$. The grayscale value at location i, say $v(i)$, is replaced by a weighted average of all the pixels in the image, say $NL(v)(i)$, defined as follows:

$$NL(v)(i) = \sum_{j \in I} a(i, j) v(j) \tag{1}$$

where $\{a(i, j)\}_{i,j}$ – all nonnegative – are similarity weights between pixels i and j and satisfy $\sum_{j \in I} a(i, j) = 1$. The similarity weights are defined as

$$a(i, j) = \frac{1}{Z(i)} e^{-\frac{\|patch(i) - patch(j)\|_2^2}{\epsilon}} \tag{2}$$

where $Z(i)$ is a normalizing factor $Z(i) = \sum_j e^{-\frac{\|patch(i)-patch(j)\|_2^2}{\epsilon}}$. The patches $patch(i)$ and $patch(j)$ are represented as column vectors. The parameter ϵ controls the decay of the exponential function. A similarity weight $a(i, j)$ is close to 1 if the patches at pixels i and j are close in L^2 sense. This method can be interpreted as follows: the set of patches is a dictionary of the image that is redundant. Any corrupted patch is then replaced by a weighted average. The noise is attenuated by the averaging. The NL-Means has proved to be one of the most efficient algorithm for de-noising. The idea of nonlocal comparison of patches has been taken up by Gilboa et al. [5] to define first a general gradient operator and then nonlocal total variation (TV) norms. The nonlocal TV norm replaces the classic TV norm in the Rudin–Osher model [14]. This approach has proven to better detect and remove irregularities from textures. However, it is not capable of separating textures which is our main goal. This is illustrated in Fig. 1 where we consider two textured images with additive Gaussian noise for which we apply the fast nonlocal TV algorithm of Bresson [2]. In the top left image of Fig. 1, one would like to separate the scarf from any other texture such as the chair in the background. The bottom left seismic image consists of several layers (e.g. vertical, horizontal) that intersect; the nonlocal methods such as NL-Means or NL-TV are incapable of separating them.

We naturally propose to adapt the regularized nonlocal operators framework to our specific target pattern problem. In that context, instead of comparing any pair of patches, we only compare patches to a reference set. We define a dictionary that consists of patches extracted from a user-specified texture. We are then able to extract the part of the image that is similar to our target texture. Several approaches are possible for building the dictionary. The first approach would be to consider an overcomplete dictionary; any patch from the reference set is an element of a large and redundant dictionary. To