



Gabor shearlets

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ABSTRACT

In this paper, we introduce Gabor shearlets, a variant of shearlet systems, which are based on a different group representation than previous shearlet constructions: they combine elements from Gabor and wavelet frames in their construction. As a consequence, they can be implemented with standard filters from wavelet theory in combination with standard Gabor windows. Unlike the usual shearlets, the new construction can achieve a redundancy as close to one as desired. Our construction follows the general strategy for shearlets. First we define group-based Gabor shearlets and then modify them to a cone-adapted version. In combination with Meyer filters, the cone-adapted Gabor shearlets constitute a tight frame and provide low-redundancy sparse approximations of the common model class of anisotropic features which are cartoon-like functions.

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1. Introduction

During the last 10 years, directional representation systems such as curvelets and shearlets were introduced to accommodate the need for sparse approximations of anisotropic features in multivariate data. These anisotropic features, such as singularities on lower dimensional embedded manifolds, called for representation systems to sparsely approximate such data. Prominent examples in the 2-dimensional setting are edge-like structures in images in the regime of explicitly given data and shock fronts in transport equations

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in the regime of implicitly given data. Because of their isotropic nature, wavelets are not as well adapted to this task as curvelets [3], contourlets [6], or shearlets [19]. Recently, a general framework for directional representation systems based on parabolic scaling – a scaling adapted to the fact that the regularity of the singularity in the considered model is C^2 – was introduced in [8] seeking to provide a comprehensive viewpoint towards sparse approximations of cartoon-like functions.

Among these representation systems, shearlets distinguished themselves by the fact that they are available as compactly supported systems – which is desirable for applications requiring high spatial localization such as PDE solvers – and also provide a unified treatment of the continuum and digital setting thereby ensuring faithful implementations. Shearlets were introduced in [9] with the early theory focusing on band-limited shearlets, see e.g. [11]. Later, a compactly supported variant was introduced in [18], which again provides optimally sparse approximations of cartoon-like functions [20]. In contrast to those properties, contourlets do not provide optimally sparse approximations and curvelets are neither compactly supported nor do they treat the continuum and digital realm uniformly due to the fact that they are based on rotations in contrast to shearing.

1.1. Key problem

One major problem – which might even be considered a “holy grail” of the area of geometric multiscale analysis – is whether a system can be designed to be

- (P1) an orthonormal basis,
- (P2) compactly supported,
- (P3) possessing a multiresolution structure,
- (P4) and providing optimally sparse approximations of cartoon-like functions.

Focusing from now on entirely on shearlets, we observe that bandlimited shearlets satisfy (P4) while replacing (P1) with being a tight frame. Compactly supported shearlets accommodate (P2) and (P4), and form a frame with controllable frame bounds as a substitute for (P1). We are still far from being able to construct a system satisfying all those properties – also by going beyond shearlets –, and it is not even clear whether this is at all possible, cf. also [17]. Several further attempts were already made in the past. In [21], shearlet systems were introduced based on a subdivision scheme, which naturally leads to (P2) and (P3), but not (P1) – not even being tight – and (P4). In [13], a different multiresolution approach was utilized leading to systems which satisfy (P2) and (P3), but not (P4), and (P1) only by forming a tight frame without results on their redundancy.

1.2. What are Gabor shearlets?

The main idea of the present construction is to use a deformation of the group operation with which common shearlet systems are generated, together with a decomposition in the frequency domain to ensure an almost uniform treatment of different directions, while modeling the systems as closely as possible after the one-dimensional multiresolution analysis (MRA) wavelets. To be more precise, the new group operation includes shears and chirp modulations which satisfy the well-studied Weyl–Heisenberg commutation relations. Thus, the shear part naturally leads us to Gabor frame constructions instead of an alternative viewpoint in which shears enter in composite dilations [10]. The filters appearing in this construction can be chosen as the trigonometric polynomials belonging to standard wavelets or to M -band versions of them, or as the smooth filters associated with Meyer’s construction. To achieve the optimal approximation rate for cartoon-like functions, we use a cone adaptation procedure. But in contrast to other constructions, we avoid incorporating redundancy in this step.

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