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Letter to the Editor

A spatio-temporal Gaussian-Conical wavelet with high aperture selectivity for motion and speed analysis

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ABSTRACT

The construction of a spatio-temporal wavelet and its tuning to speed was first realized in the 90s on the Morlet wavelet by Duval-Destin (1991, 1993) [14,15]. This enabled to demonstrate the capacities of the speed-tuned Morlet for psychovisual analysis. This construction was also used very efficiently in a powerful aerial target tracking algorithm by Mujica et al. (1999, 2000) [20,21]. In the last decade, this tool was proposed as an elegant and efficient alternative framework to the Optical Flow (OF), the Block Matching (BM) or the phase difference, for the study of motion estimation in image sequences. Nevertheless, the aperture selectivity of the $2D+T$ Morlet wavelet presents some difficulties. Here we propose to replace the 2D Morlet wavelet by a Gaussian-Conical (GC) wavelet for the spatial part of the spatio-temporal wavelet, since the GC wavelet has a better aperture selectivity and allows a very simple adjustment of the aperture. Therefore we build a new, highly directional, speed-tuned wavelet called Gaussian-Conical–Morlet (GCM) wavelet. Like the speed-tuned $2D+T$ Morlet, the new wavelet presents very good characteristics in motion estimation and tracking, namely long temporal dependence, robustness to noise and to occlusions, and supersedes the OF (Optical Flow) and BM (Block Matching) techniques. However, for aperture selectivity, directional speed-capture and spectral recognition and tracking, GCM easily outperforms Morlet. This paper describes the GCM construction, utilization and aperture performances.

1. Introduction

The continuous wavelet transform has proved to be a very efficient tool for signal analysis. In the late 80s and the 90s, developments to adapt the wavelet transform to various motions have been proposed by Duval-Destin and Murenzi [15].

The group of analysis parameters, i.e., usually position, scale and rotation, has been extended to speed, acceleration and deformation. This has led to various types of time dependent wavelets [6, Chapter 10]. Then a very performant algorithm for missile tracking was set up by Mujica et al. [20,21] using such wavelets (see also [6, Chapter 10]). The same energy-density based algorithm was used by Hong et al. [16] and by Wang et al. [23], but with Expectation–Maximization plus Gaussian mixture approach and scale functional relation plus ST processing blocks. Later, in [24], the latter authors also use image transformation to time-varying $(1D+T)$ signals.

The advantage of velocity detection with the motion-tuned spatio-temporal CWT over other known methods, like Optical Flow (OF) [7], Block Matching (BM) and phase difference has been already discussed in [8,9]. These methods work on the

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motion of pixels or of blocks, but not on regions or objects. They assume that the object is constant from frame to frame and that the *object signature* does not change with time. They are not inherently scalable either. Like BM, OF has a short time dependence, which is not very accurate for slow motion or trajectory estimation. The four characteristics of object tracking, by spectral signature or spatial scale, long temporal dependence, robustness to noise and robustness to occlusions, are the strength of wavelet analysis, and we plan to show it further for pertinent feature extraction and recognition in sequence analysis, for object tracking, for video compression and for video data mining.

It is standard practice in wavelet analysis to use the *continuous* WT for the *analysis* of signals and the *discrete* WT for reconstruction, for instance, after noise filtering. The rationale is that the values of the parameters to be measured are not known in advance, so that a discretization chosen *a priori* (e.g. dyadic) may fail to detect the fine details. Of course, the CWT is discretized for the implementation, but the grid is arbitrary. Thus, in our case, the (discretized) CWT is the technique to use, as in all previous works on the subject [6,15,20,21].

Because of its good localization both in position space and in frequency space, but also because the symmetry of its envelope, the Morlet wavelet was first chosen to be tuned to speed. In Fourier space, this wavelet reads as

$$\hat{\psi}_M(\vec{k}) = \sqrt{\epsilon}\left(\exp\left(-\frac{1}{2}\left|A^{-1}(\vec{k}-\vec{k}_0)\right|^2\right) - \exp\left(-\frac{1}{2}\left|A^{-1}\vec{k}_0\right|^2\right)\exp\left(-\frac{1}{2}\left|A^{-1}\vec{k}\right|^2\right)\right), \tag{1.1}$$

where $A = \mathrm{diag}[1, \epsilon^{-1/2}]$, $\epsilon \geqslant 1$, is a 2×2 anisotropy matrix and the correction term, which ensures admissibility of the wavelet, is usually dropped (see [6, Eq. (3.18)]), since it is negligible for practical values of the parameters.

Nevertheless numerous difficulties remain when using this wavelet in directional analysis. Although it has a good capability for directional filtering, its aperture selectivity is poor. It is *directional* in the sense defined in [4] and [6, Section 3.3], namely, *"A wavelet ψ is said to be directional if the effective support of its Fourier transform $\hat{\psi}$ is contained in a convex cone in spatial frequency space"*, but the anisotropy parameter $\epsilon > 1$ is needed in order to get a decent angular selectivity. In addition, the Morlet wavelet has a major drawback: its angular selectivity increases with the length of the wave vector $\vec{k}_0$, since the support cone gets narrower, but at the same time the amplitude decreases as $\exp(-|\vec{k}_0|^2)$.

In order to achieve a more efficient directional wavelet, a better method is to consider a smooth function with support in a strictly convex cone $\mathcal{C}$ and behaving inside this cone as $P(\vec{k})e^{-\vec{\zeta}\cdot\vec{k}}$ where $\vec{\zeta} \in \mathcal{C}$ and $P(\cdot)$ is a polynomial. This leads to the conical wavelets, in particular the Cauchy wavelet and the Gaussian-Conical wavelet, if the exponential is replaced by a Gaussian [4,6]. These are genuine directional wavelets that don't suffer from the defects of the Morlet wavelet. We have used these wavelets as a basic ingredient for a new construction of motion-tuned, and in particular speed-tuned wavelets. The development of these wavelets and their use in motion analysis is the aim of the present paper.

2. Preliminaries: the 2D continuous WT

In order to motivate our construction and to fix notations, we begin with a brief reminder of the 2D continuous wavelet transform (CWT), following [6, Chapter 2]. A 2D wavelet is a function $\psi \in L^2(\mathbb{R}^2, \mathrm{d}\vec{x})$ satisfying the admissibility condition

$$c_\psi \equiv (2\pi)^2 \int_{\mathbb{R}^2} \mathrm{d}\vec{k}\, \frac{|\hat{\psi}(\vec{k})|^2}{|\vec{k}|^2} < \infty, \tag{2.1}$$

where $\hat{\psi}$ is the Fourier transform of ψ. In practice, this condition is often replaced by the slightly weaker one

$$\hat{\psi}(\vec{0}) = 0 \quad \Longleftrightarrow \quad \int_{\mathbb{R}^2} \mathrm{d}\vec{x}\, \psi(\vec{x}) = 0. \tag{2.2}$$

Given a 2D signal (an image) $s \in L^2(\mathbb{R}^2)$, its CWT with respect to the wavelet ψ is given by the inner product

$$\begin{aligned} W_\psi s(\vec{b}, a, \theta) &= \langle \psi_{\vec{b},a,\theta} | s \rangle \\ &= a^{-1} \int_{\mathbb{R}^2} \mathrm{d}\vec{x}\, \overline{\psi\left(a^{-1} r^{-\theta}(\vec{x}-\vec{b})\right)} s(\vec{x}), \qquad (2.3) \\ &= a \int_{\mathbb{R}^2} \mathrm{d}\vec{k}\, e^{i\vec{b}\cdot\vec{k}}\, \overline{\hat{\psi}\left(a r^{-\theta}\vec{k}\right)} \hat{s}(\vec{k}), \qquad (2.4) \end{aligned}$$

where the overbar denotes complex conjugation, $\psi_{\vec{b},a,\theta}$ is a copy of ψ translated by $\vec{b} \in \mathbb{R}^2$, dilated by a factor $a > 0$, and rotated by an angle $\theta \in [0, 2\pi]$, that is,

$$\psi_{\vec{b},a,\theta} = a^{-1}\psi\left(a^{-1} r_\theta^{-1}(\vec{x}-\vec{b})\right), \tag{2.5}$$

where r^θ is the familiar 2×2 rotation matrix of angle θ