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The lower and upper bounds of the first eigenvalues for the bi-Laplace operator on Finsler manifolds ☆

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ABSTRACT

In this paper, we will estimate the lower and upper bounds of the first eigenvalues for bi-Laplace operators on Finsler manifolds.

1. Introduction

Recently, Finsler geometry attracts much attention, since it has broader applications in nature science. And, the investigate of the first eigenvalue of Laplace in Finsler manifolds plays an important role in Finsler geometry, there are a lot of results, such as [5,14,16], etc.

Bi-Laplace eigenvalue problems on compact Riemannian manifold are very interesting, such as *clamped plate problem* and *buckling problem*, these two problems are described as follows, let M be an n-dimensional compact connected Riemannian manifold with smooth boundary ∂M. The following two eigenvalue problems

$$\begin{cases} \Delta^2 u = \Gamma u \text{ in } \ \mathrm{M}, \\ u|_{\partial\Omega} = \frac{\partial u}{\partial \vec{n}}|_{\partial M} = 0 \end{cases}$$

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and

$$\begin{cases}\Delta^2 u = -\Lambda\Delta u \text{ in } \mathrm{M}, \\ u|_{\partial\Omega} = \frac{\partial u}{\partial \vec{n}}|_{\partial M} = 0\end{cases}$$

are called *clamped plate problem* and *buckling problem*, where Δ^2 is the bi-harmonic operator and $\vec{n}$ denotes the outer unit normal vector field of the boundary ∂M. For researches on these two problems, we refer the readers to [1,4,6,7,11], etc.

Similarly, let $(M, F, \mathrm{d}\mu)$ be an n-dimensional compact connected Finsler manifold with smooth boundary ∂M. We also called the following two eigenvalue problems are *clamped plate problem* and *buckling problem* on Finsler manifold:

$$\begin{cases}\Delta^{\nabla u}\Delta u = \Gamma u \quad \text{in} \quad \mathrm{M}_u, \\ u|_{\partial\Omega} = g_{\vec{n}}(\vec{n}, \nabla u)|_{\partial M} = 0\end{cases} \tag{1.1}$$

and

$$\begin{cases}\Delta^{\nabla u}\Delta u = -\Lambda\Delta u \quad \text{in} \quad \mathrm{M}_u, \\ u|_{\partial\Omega} = g_{\vec{n}}(\vec{n}, \nabla u)|_{\partial M} = 0,\end{cases} \tag{1.2}$$

where Δ and $\Delta^{\nabla u}$ are Laplacian and weighted Laplacian, $\vec{n}$ denotes the outer unit normal vector field of the boundary ∂M and $g_{\vec{n}}$ denotes the induced Riemannian structure on ∂M.

Recently, in [3], Chen, Cheng, Wang and Xia have investigated the first eigenvalues of *clamped plate problem* and *buckling problem* on n-dimensional compact connected Riemannian manifolds, and under the assumption that the Ricci curvature of M is bounded from below by $(n-1)$. They got the lower bounds

$$\Gamma_1 > n\lambda_1, \quad \Lambda_1 > n,$$

where λ_1 denotes the first eigenvalue of the Dirichlet eigenvalue problem. These interesting results inspire us to study the lower and upper bounds of the first eigenvalues for the problems (1.1) and (1.2).

This paper is organized as follows. In Section 2, we briefly recall fundamentals in Finsler geometry. In Section 3, we will estimate the lower and upper bounds of the first eigenvalues for the problems (1.1) and (1.2).

2. Preliminaries

In this section, we will briefly review the fundamentals of Finsler geometry, for which we refer to [2,10,12].

2.1. Finsler structure and Chern connection

Definition 2.1. Let M be an n-dimensional smooth manifold. We say a function $F : TM \to [0, \infty)$ is a *Finsler structure* if the following three conditions hold:
(i) F is C^∞ on $TM\backslash 0$;
(ii) $F(\lambda V) = \lambda F(V)$ for all $V \in TM\backslash 0$ and $\lambda > 0$;
(iii) For any $V \in T_x M\backslash 0$, the $n \times n$ matrix

$$(g_{ij}(V))_{ij=1}^n = \left(\frac{1}{2}[F^2(V)]_{V^i V^j}\right)_{ij=1}^n$$

is positive definite. The pair (M, F) is called a Finsler manifold.