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Erratum

Erratum to: "On biminimal submanifolds in nonpositively curved manifolds" [Differ. Geom. Appl. 35 (2014) 1–8]

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ABSTRACT

We correct theorems of Luo (2014) [1], concerning nonexistence of complete biminimal submanifolds in nonpositive curvature space forms, and Lemma 4.2 in Luo (2014) [1].

In [1], the present author gave several results concerning the nonexistence of complete biminimal submanifolds in nonpositive curvature space forms. The main theorems in [1] are Theorems 1.3, 1.4 and 1.5. In the proof of Theorem 1.3 in [1] we used the inequality $b_n \leq \frac{1}{2}b_{2n}$ of page 7, line 3 to get the inequality of page 7, line 7. But this is wrong, because to get the inequality of page 7, line 7, the inequality we need is $b_{2n} \leq \frac{1}{2}b_n$. In this note we give a proof of Theorem 1.3 (also Theorems 1.4, 1.5) by adding an additional condition.

Theorem 0.1. *Let $f : (M, g) \to (N, \langle , \rangle)$ be a complete positive (that is $\lambda > 0$) biminimal submanifold (resp. hypersurface). Assume that the sectional curvature (resp. Ricci curvature) of $(N, \langle , \rangle)$ is nonpositive and $\int_{B_\rho(x_0)} |\vec{H}|^{p+2} d\mu_g$ is of at most polynomial growth of ρ, then f is minimal.*

Here and in the following $B_\rho(x_0)$ denotes the geodesic ball on M centered at x_0 of radius ρ. We say a function $f : \mathbb{R}^+ \to \mathbb{R}^+$ is of at most polynomial growth, if as $\rho \to \infty$, $f(\rho) \leq C(1 + \rho^s)$ for some positive constant C independent of ρ and s a positive integer.

In particular, as a direct corollary of Theorem 0.1 we have

Theorem 0.2. *Any complete positive biminimal submanifold in a Euclidean space which satisfies that $\int_{B_\rho(x_0)} |\vec{H}|^{p+2} d\mu_g$ is of at most polynomial growth of ρ is minimal.*

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Furthermore for negative biminimal submanifolds in negative space forms $N(c)$ of sectional curvature $c \le 0$ we have

Theorem 0.3. *Let $f : (M^m, g) \to N^n(c)$ be a complete biminimal submanifolds in a space form $N(c)$ with constant sectional curvature $c \le 0$ and assume that $\int_{B_\rho(x_0)} |\vec{H}|^{p+2} d\mu_g$ is of at most polynomial growth of ρ. If $\lambda > mc$, f is minimal.*

Proof of Theorem 0.1. From Eq. (1.7) in [1] we see that

$$\begin{aligned}\Delta|\vec{H}|^2 &= 2|\nabla^\perp \vec{H}|^2 + 2\langle \vec{H}, \Delta^\perp \vec{H}\rangle + 2\lambda|\vec{H}|^2 \\ &= 2|\nabla^\perp \vec{H}|^2 + 2\sum_{i=1}^m \langle h(A_{\vec{H}} e_i, e_i), \vec{H}\rangle - 2\sum_{i=1}^m \langle R^N(e_i, \vec{H})e_i, \vec{H}\rangle + 2\lambda|\vec{H}|^2 \\ &= 2|\nabla^\perp \vec{H}|^2 + 2\sum_{i=1}^m \langle A_{\vec{H}} e_i, A_{\vec{H}} e_i\rangle - 2\sum_{i=1}^m \langle R^N(e_i, \vec{H})e_i, \vec{H}\rangle + 2\lambda|\vec{H}|^2.\end{aligned}$$

Obviously if N has non-positive sectional curvature, $-\sum_{i=1}^m \langle R^N(e_i, \vec{H})e_i, \vec{H}\rangle \ge 0$. Furthermore, if M is a hypersurface, we see that $-\sum_{i=1}^m \langle R^N(e_i, \vec{H})e_i, \vec{H}\rangle = -Ric^N(\vec{H}, \vec{H}) \ge 0$. Therefore

$$\begin{aligned}\Delta|\vec{H}|^2 &\ge 2|\nabla^\perp \vec{H}|^2 + 2\sum_{i=1}^m \langle A_{\vec{H}} e_i, A_{\vec{H}} e_i\rangle + 2\lambda|\vec{H}|^2 \\ &\ge 2|\nabla^\perp \vec{H}|^2 + \frac{2}{m}|\vec{H}|^4 + 2\lambda|\vec{H}|^2, \end{aligned} \tag{0.1}$$

where in the last inequality we used $\sum_{i=1}^m \langle A_{\vec{H}} e_i, A_{\vec{H}} e_i\rangle \ge \frac{1}{m}|\vec{H}|^4$, which can be seen as follows: Let $x \in M$, when $\vec{H}(x) = 0$, we are done. If $\vec{H}(x) \ne 0$, set $e_{m+t} = \frac{\vec{H}}{|\vec{H}|}$, then $\vec{H}(x) = H_{m+t}(x)e_{m+t}$ and $|\vec{H}|^2 = H_{m+t}^2$. Hence we have at x,

$$\begin{aligned}\sum_{i=1}^m \langle A_{\vec{H}} e_i, A_{\vec{H}} e_i\rangle &= H_{m+t}^2 \sum_{i=1}^m \langle A_{e_{m+t}} e_i, A_{e_{m+t}} e_i\rangle \\ &= H_{m+t}^2 |B_{m+t}|_h^2 \\ &\ge \frac{1}{m} H_{m+t}^4 \\ &= \frac{1}{m}|\vec{H}|^4.\end{aligned}$$

Let $\gamma : M \to \mathbb{R}^+$ be a cut off function such that

$$\gamma = 1 \text{ on } B_\rho, \gamma = 0 \text{ on } M \setminus B_{2\rho}, \text{ and } |\nabla\gamma| \le \frac{C}{\rho},$$

for some constant C independent of ρ. Here B_ρ is a geodesic ball of radius ρ on M. Then

$$\begin{aligned}&-\int_M \nabla|\vec{H}|^2 \nabla(|\vec{H}|^a \gamma^2) d\mu_g \\ &= \int_M \Delta|\vec{H}|^2 |\vec{H}|^a \gamma^2 d\mu_g\end{aligned}$$