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Fiber product preserving bundle functors of vertical type

J. Kurek [a,*], W.M. Mikulski [b]

[a] Institute of Mathematics, UMCS Lublin, pl. M. Curie Sklodowska 1, Lublin, Poland
[b] Institute of Mathematics, Jagiellonian University, Łojasiewicza 6, 30 348 Kraków, Poland

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ABSTRACT

We extend the concept of vertical Weil functors V^A corresponding to Weil algebras A to the one of generalized vertical Weil functors $V^{\mathcal{A}}$ on $\mathcal{FM}_m$ corresponding to Weil algebra bundle functors $\mathcal{A}$ on $\mathcal{M}f_m$. Next, we show that the fiber product preserving bundle functors F on $\mathcal{FM}_m$ of vertical type are the generalized vertical Weil functors $V^{\mathcal{A}}$ on $\mathcal{FM}_m$. That F is of vertical type it means that $F_x f : F_x Y \to F_x Y^1$ depends only on $f_x : Y_x \to Y_x^1$ for any $\mathcal{FM}_m$ objects Y and Y_1 with the same base and any $\mathcal{FM}_m$-map $f : Y \to Y^1$ with the identity as the base map.

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0. Introduction

All manifolds are assumed to be without boundaries, second countable, Hausdorff and smooth, and maps to be of $\mathcal{C}^\infty$.

The general concept of bundle functors (and their natural transformations) on a local category $\mathcal{C}$ over manifolds can be found in Sect. 18 in [6]. We need this concept in the case $\mathcal{C} = \mathcal{M}f_m$ or $\mathcal{C} = \mathcal{FM}_m$, where $\mathcal{M}f_m$ is the category of all m-dimensional manifolds and their embeddings and $\mathcal{FM}_m$ is the category of all fibred manifolds with m-dimensional bases and all fibred maps with embeddings as base maps. Thus a bundle functor F on $\mathcal{FM}_m$ is a functor $F : \mathcal{FM}_m \to \mathcal{FM}$ such that the value FY of Y is a fibred manifold $\pi_Y : FY \to Y$ for any $\mathcal{FM}_m$-object $p : Y \to M$, the value $Ff : FY \to FY^1$ of $f : Y \to Y^1$ is a fiber map covering f for any $\mathcal{FM}_m$-map $f : Y \to Y^1$, and $Fi_U : FU \to \pi_Y^{-1}U$ is a diffeomorphisms for the inclusion map $i_U : U \to Y$ of an open subset U of Y. The definition of bundle functors on $\mathcal{M}f_m$ is quite similar (we replace $\mathcal{FM}_m$ by $\mathcal{M}f_m$).

* Corresponding author.
E-mail addresses: kurek@hektor.umcs.lublin.pl (J. Kurek), Wlodzimierz.Mikulski@im.uj.edu.pl (W.M. Mikulski).

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A bundle functor F on $\mathcal{FM}_m$ is of vertical type if for any $\mathcal{FM}_m$-objects Y and Y^1 with the same basis M, any $x_o \in M$ and any $\mathcal{FM}_m$-map $f : Y \to Y_1$ covering the identity of M the restriction $F_{x_o} f : F_{x_o} Y \to F_{x_o} Y^1$ of Ff depends on $f_{x_o} : Y_{x_o} \to Y^1_{x_o}$, only.

A bundle functor F on $\mathcal{FM}_m$ is fiber product preserving if $F(Y \times_M Y^1) \tilde{=} FY \times_M FY^1$ for any $\mathcal{FM}_m$-objects Y and Y^1 with the same base M.

A natural transformation $\eta : F \to F^1$ between bundle functors on $\mathcal{FM}_m$ is a family of maps $\eta_Y : FY \to F^1 Y$ for any $\mathcal{FM}_m$-manifolds Y such that $F^1 f \circ \eta_Y = \eta_{Y^1} \circ Ff$ for any $\mathcal{FM}_m$-map $f : Y \to Y^1$. (One can show that then $\eta_Y : FY \to F^1 Y$ is a fibred map covering the identity map id_Y for any $\mathcal{FM}_m$-manifold Y [6].)

A Weil algebra is a finite dimensional real local commutative algebra A with unity (i.e. $A = \mathbf{R}.1 \oplus N_A$, where N_A is a finite dimensional ideal of nilpotent elements).

In [9], A. Weil introduced the concept of near A points on a manifold M as an algebra homomorphisms of the algebra $C^\infty(M, \mathbf{R})$ of smooth functions on M into a Weil algebra A. Nowadays the space $T^A M$ of all near A-points on M is called a Weil bundle. D. Eck (see [2]), O.O. Luciano (see [8]) and G. Kainz and P.W. Michor (see [4]) proved independently that the product preserving bundle functors G on the category $\mathcal{M}f$ of all manifolds and maps (i.e. $G(M \times M^1) \tilde{=} GM \times GM^1$ for all manifolds M and M^1) are the Weil functors T^A for Weil algebras $A = G\mathbf{R}$, and that the natural transformations $\mu : G \to G^1$ between product preserving bundle functors on $\mathcal{M}f$ are in bijection with the algebra homomorphisms $\mu_\mathbf{R} : G\mathbf{R} \to G^1\mathbf{R}$.

A Weil algebra bundle functor on $\mathcal{M}f_m$ is a bundle functor $A : \mathcal{M}f_m \to \mathcal{FM}$ such that $A_x M$ is a Weil algebra and $A_x f : A_x M \to A_{f(x)} N$ is an algebra isomorphism for any $\mathcal{M}f_m$-map $f : M \to N$ between m-manifolds M and N and any point $x \in M$ (or shortly and more precisely, A is a bundle functor from $\mathcal{M}f_m$ into the category of all Weil algebra bundles and their algebra bundle maps).

A natural transformation between Weil algebra bundle functors A and A^1 on $\mathcal{M}f_m$ is a natural transformation $\varphi : A \to A^1$ between bundle functors such that $(\varphi_M)_x : A_x M \to A^1_x M$ is an algebra homomorphism for any m-manifold M and any point $x \in M$.

We have the following examples of Weil algebra bundle functors on $\mathcal{M}f_m$.

(i) The trivial Weil algebra bundle functor A on $\mathcal{M}f_m$ given by $AM = M \times A$ and $Af = f \times id_A$, where A is a fixed Weil algebra.
(ii) The Weil algebra bundle functor A on $\mathcal{M}f_m$ given by $AM = (\bigwedge TM)^0$ and $Af = \bigwedge Tf_{|(\bigwedge TM)^0}$, where $\bigwedge TM = (\bigwedge TM)^0 \oplus (\bigwedge TM)^1 = \bigcup_{x \in M} \bigwedge T_x M$ is the Grassmann algebra bundle of the tangent bundle TM and $(\bigwedge TM)^0$ is the even degree subalgebra bundle.
(iii) In the previous example we can replace the tangent functor T by an arbitrary vector bundle functor G on $\mathcal{M}f_m$.
(iv) The Weil algebra bundle functor A on $\mathcal{M}f_m$ given by $AM = J^r(M, \mathbf{R})$ and $Af = J^r(f, id_\mathbf{R})$.
(v) We can apply fibre-wise tensor product to the above examples of Weil algebra bundle functors on $\mathcal{M}f_m$.

In Example 1 of the present note, given a Weil algebra bundle functor A on $\mathcal{M}f_m$, we construct canonically a fiber product preserving bundle functor V^A on $\mathcal{FM}_m$ of vertical type by

$$V^A Y = \bigcup_{x \in M} T^{A_x M} Y_x.$$

We call it the generalized vertical Weil functor corresponding to A.

In this way, we obtain a functor

$$\mathcal{V} : \mathcal{WABF}_m \to \mathcal{VFPPBF}_m$$

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