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Journal of Approximation Theory 213 (2017) 1–22

Journal of Approximation Theory

www.elsevier.com/locate/jat

Full length article

C_0-semigroups and resolvent operators approximated by Laguerre expansions

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Received 11 March 2015; received in revised form 27 July 2016; accepted 27 September 2016
Available online 14 October 2016

Communicated by Martin Buhmann

To Laura and Pablo

Abstract

In this paper we introduce Laguerre expansions to approximate vector-valued functions. We apply this result to approximate C_0-semigroups and resolvent operators in abstract Banach spaces. We study certain Laguerre functions in order to estimate the rate of convergence of these expansions. Finally, we illustrate the main results of this paper with some examples: shift, convolution and holomorphic semigroups, where the rate of convergence is improved.
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MSC: primary 33C45; 47D06; secondary 41A35; 47A60

Keywords: Laguerre expansions; C_0-semigroups; Resolvent operators; Functional calculus

1. Introduction

Representations of functions through series of orthogonal polynomials such as Legendre, Hermite or Laguerre are well known in mathematical analysis and applied mathematics. They

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http://dx.doi.org/10.1016/j.jat.2016.09.001
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allow us to approximate functions by series of orthogonal polynomials using different types of convergence: a pointwise way, uniformly, or in Lebesgue norm. Two classical monographs where we can find this kind of results are [21, Chapter 4] and [28, Chapter IX]. In this paper we are concentrated on Laguerre expansions.

Rodrigues' formula gives a representation of generalized Laguerre polynomials,

$$L_n^{(\alpha)}(t) = e^t \frac{t^{-\alpha}}{n!} \frac{d^n}{dt^n}(e^{-t} t^{n+\alpha}), \quad t \in \mathbb{R},$$

for $\alpha \in \mathbb{R}$ and $n \in \mathbb{N} \cup \{0\}$. The following theorem, which appears in [21, Sec. 4.23] and whose statement was originally proved by J.V. Uspensky in [30], gives a pointwise approximation for scalar functions in terms of Laguerre polynomials. As we prove in Theorem 3.3, this result also holds for vector-valued functions in abstract Banach spaces.

Theorem 1.1. *Let $\alpha > -1$ and $f : (0, \infty) \to \mathbb{C}$ be a differentiable function such that the integral $\int_0^\infty e^{-t} t^\alpha |f(t)|^2\, dt$ is finite. Then the series $\sum_{n=0}^\infty c_n(f) L_n^{(\alpha)}(t)$ converges pointwise to $f(t)$ for $t > 0$, where*

$$c_n(f) := \frac{n!}{\Gamma(n + \alpha + 1)} \int_0^\infty e^{-t} t^\alpha f(t) L_n^{(\alpha)}(t)\, dt.$$

There exists a large amount of results about Laguerre expansions: for example, Laguerre expansions of analytic functions are considered in [26]; the decay of coefficients is also studied in [31] in connection with the other properties of the function given by their Laguerre expansions; and the algebraic structure related to the Laguerre expansions is covered in detail in [19].

In particular, we apply Theorem 1.1 to the function e_a (where $e_a(t) := e^{-at}$) in order to show that

$$e^{-at} = \sum_{n=0}^\infty \frac{a^n}{(a+1)^{n+\alpha+1}} L_n^{(\alpha)}(t), \quad a > 0, \tag{1.1}$$

and the Laguerre expansion converges pointwise for $t > 0$.

Through Laguerre polynomials, Laguerre functions are defined by

$$\mathscr{L}_n^{(\alpha)}(t) := \sqrt{\frac{n!}{\Gamma(n + \alpha + 1)}}\, t^{\frac{\alpha}{2}} e^{-\frac{t}{2}} L_n^{(\alpha)}(t), \quad t > 0,$$

for $\alpha > -1$. These functions form an orthonormal basis in the Hilbert space $L^2(\mathbb{R}_+)$. Furthermore, let f be in $L^p(\mathbb{R}_+)$, $\frac{4}{3} < p < 4$, and $a_k(f) := \int_0^\infty \mathscr{L}_k^{(\alpha)}(t) f(t)\, dt$ for $k \in \mathbb{N} \cup \{0\}$. Then $\|S_n(f) - f\|_p \to 0$ as $n \to \infty$, with

$$S_n(f)(t) := \sum_{k=0}^n a_k(f) \mathscr{L}_k^{(\alpha)}(t), \quad t > 0,$$

see [3, Theorem 1].

On the other hand, a C_0-semigroup $(T(t))_{t\geq 0}$ is a one parameter family of linear and bounded operators on a Banach space X which may be interpreted, approximately, as $(e^{-tA})_{t\geq 0}$. The (densely defined) operator $-A$, defined by

$$-Ax := \lim_{t\to 0^+} \frac{T(t)x - x}{t}, \quad \text{when the limit exists}, \; x \in D(A),$$

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