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Full length article

Tractability of multivariate problems for standard and linear information in the worst case setting: Part I

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Abstract

We present a lower error bound for approximating linear multivariate operators defined over Hilbert spaces in terms of the error bounds for appropriately constructed linear functionals as long as algorithms use function values. Furthermore, some of these linear functionals have the same norm as the linear operators. We then apply this error bound for linear (unweighted) tensor products. In this way we use negative tractability results known for linear functionals to conclude the same negative results for linear operators. In particular, we prove that L_2-multivariate approximation defined for standard Sobolev space suffers the curse of dimensionality if function values are used although the curse is not present if linear functionals are allowed.
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1. Introduction

The understanding of the intrinsic difficulty of approximation of d-variate problems is a challenging problem especially when d is large. We consider algorithms that approximate d-variate

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problems and use finitely many linear functionals: we compare the class $\Lambda^{\rm all}$ of arbitrary linear information functionals with the class $\Lambda^{\rm std}$ of information functionals that are given by function evaluations at single points.

To find best algorithms for the class $\Lambda^{\rm all}$ is usually much easier than for the class $\Lambda^{\rm std}$, in particular if the source space is a Hilbert space. This is especially the case for the worst case setting. The state of art may be found in [9], where the reader may find a number of surprising results. For example, there are multivariate problems for which the best rate of convergence of algorithms using n appropriately chosen linear functionals is $n^{-1/2}$ whereas for n function values the best rate can be arbitrarily bad, i.e., like $1/\ln(\ln(\cdots\ln(n)))$, where the number of ln can be arbitrarily large, see [4] which is also reported in [9, pp. 292–304]. Furthermore, the dependence on d may be quite different for the linear and standard classes. There are examples of interesting multivariate problems for which the dependence on *d is not* exponential for the class $\Lambda^{\rm all}$, and *is* exponential for the class $\Lambda^{\rm std}$. The exponential dependence on d is called the *curse of dimensionality*. On the other hand, for some other multivariate problems there is no difference between $\Lambda^{\rm all}$ and $\Lambda^{\rm std}$. Examples can be found, in particular, in [2,6,7,9].

Tractability deals with how the intrinsic difficulty of a multivariate problem depends on d and on ε^{-1}, where ε is an error threshold. We would like to know when the curse of dimensionality holds and when we have a specific dependence on d which is not exponential. There are various ways of measuring the lack of exponential dependence and that leads to different notions of tractability. In particular, we have polynomial tractability (PT) if the intrinsic difficulty is polynomial in both d and ε^{-1}. We have quasi-polynomial tractability (QPT) if the intrinsic difficulty is at most proportional to $\varepsilon^{-t\,\ln d}$ for some t independent of d and ε.

Obviously, tractability may depend on which of the classes $\Lambda^{\rm std}$ or $\Lambda^{\rm all}$ is used. Tractability results for $\Lambda^{\rm std}$ cannot be better than for $\Lambda^{\rm all}$. The main question is for which multivariate problems they are more or less the same or for which multivariate problems they are essentially different.

These questions were already addressed in [6,7,9]. Still, especially the worst case setting is not fully understood. We would like to get a better understanding how the power of the standard class $\Lambda^{\rm std}$ is related to the power of the class $\Lambda^{\rm all}$ of information. Ideally, we would like to characterize for which multivariate problems the classes $\Lambda^{\rm std}$ and $\Lambda^{\rm all}$ lead to more or less the same tractability results and for which tractability results are essentially different.

We plan to write a number of papers about this problem under the same title. We present the first part of this project. We restrict ourselves to linear multivariate problems defined as approximation of a linear continuous operator $S : F \to G$ for general Hilbert spaces F and G. Since we want to study the class $\Lambda^{\rm std}$ we need to assume that function values are well defined and they correspond to linear continuous functionals. This is equivalent to assuming that F is a reproducing kernel Hilbert space.

For the worst case setting and for the class $\Lambda^{\rm all}$, it is known what is the best way to approximate S. The intrinsic difficulty of approximating S is defined as the *information complexity* which is the minimal number of linear functionals which are needed to find an algorithm whose worst case error is at most $\varepsilon\|S\|$. This depends on the eigenvalues of the operator $S^*S : F \to F$. For the class $\Lambda^{\rm std}$ the situation is much more complex and the *information complexity*, which is now the minimal number of function values needed to get an error $\varepsilon\|S\|$, depends not only on the eigenvalues of S^*S.

Our first result is the construction of continuous linear functionals I which are at most as hard to approximate as S for the class $\Lambda^{\rm std}$. Furthermore, we characterize I for which $\|I\| = \|S\|$.

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