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Journal of Approximation Theory 207 (2016) 380–401

Journal of Approximation Theory

www.elsevier.com/locate/jat

Full length article

Approximation order and approximate sum rules in subdivision

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Received 27 February 2015; received in revised form 21 December 2015; accepted 16 February 2016
Available online 24 March 2016

Communicated by Ulrich Reif

Abstract

Several properties of stationary subdivision schemes are nowadays well understood. In particular, it is known that the polynomial generation and reproduction capability of a stationary subdivision scheme is strongly connected with sum rules, its convergence, smoothness and approximation order. The aim of this paper is to show that, in the non-stationary case, exponential polynomials and approximate sum rules play an analogous role of polynomials and sum rules in the stationary case. Indeed, in the non-stationary univariate case we are able to show the following important facts: (i) reproduction of N exponential polynomials implies approximate sum rules of order N; (ii) generation of N exponential polynomials implies approximate sum rules of order N, under the additional assumption of asymptotical similarity and reproduction of one exponential polynomial; (iii) reproduction of an N-dimensional space of exponential polynomials and asymptotical similarity imply approximation order N; (iv) the sequence of basic limit

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http://dx.doi.org/10.1016/j.jat.2016.02.014

functions of a non-stationary scheme reproducing one exponential polynomial converges uniformly to the basic limit function of the asymptotically similar stationary scheme.

MSC: 65D17; 65D15; 41A25; 41A10; 41A30

Keywords: Subdivision schemes; Exponential polynomial generation and reproduction; Asymptotical similarity; Approximate sum rules; Approximation order

1. Introduction

In this paper we investigate theoretical properties of non-stationary subdivision schemes. In particular, we study their approximation order and the role played by approximate sum rules, a non-stationary extension of the well-known notion of sum rules. The obtained results allow us to point out similarities and differences between the stationary and non-stationary cases.

For unfamiliar readers, we briefly introduce subdivision schemes as efficient tools to design smooth curves and surfaces out of sequences of initial points. As a matter of fact, a subdivision curve or surface is obtained as the limit of an iterative procedure based on the repeated application of local refinement rules generating denser and denser sets of points starting from a coarse initial set roughly describing the desired limit shape [3,20,42]. In practical use, however, only a limited number of iterations are needed. As a consequence, subdivision schemes are very efficient if compared with traditional parametric curve and surface representations. They also stand out for ease of implementation and versatility in building free-form surfaces of arbitrary topological genus. All these advantages are the reasons for the overwhelming development of subdivision methods and their increasing use in many application areas such as computer-aided geometric design [20,42], curve and surface reconstruction [35], wavelets and multiresolution analysis [9], signal/image processing [16,39], computer games and animation [17]. Within the variety of subdivision methods studied in the literature, the class of non-stationary subdivision schemes is currently receiving great attention. This is due to the fact that non-stationary subdivision schemes are general and flexible enough to overcome the restricted capabilities of stationary subdivision schemes. As an example, we can think of the fact that stationary subdivision schemes are not capable of representing conic sections or, in general, exponential polynomials. On the contrary, non-stationary schemes can also generate exponential polynomials or exponential B-splines, that is piecewisely defined exponential polynomials [1,6,12,21,26,30,33,34,37,38]. Reproduction of piecewise exponential polynomials is important in several applications, e.g., in biomedical imaging, in geometric design and in isogeometric analysis. Moreover, non-stationary subdivision schemes include Hermite subdivision schemes. Hermite subdivision schemes are iterative methods mapping, at each iteration, a set of vector data consisting of functional values and associated derivatives, to a denser set of vector data of the same type [14,32]. They are applied in geometric modeling for the construction of curves and surfaces out of points and directional derivatives, and have recently found application in other contexts such as, for example, in the design of one-step numerical methods for the numerical solution of ODE Initial Value Problems.

The main goal of this paper is to investigate the approximation order of non-stationary subdivision schemes and the role played by approximate sum rules, a non-stationary extension of the well-known notion of sum rules. Approximate sum rules allow us to link the response