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On approximation properties of generalized Kantorovich-type sampling operators

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Abstract

In this paper, we generalize the notion of Kantorovich-type sampling operators using the Fejér-type singular integral. By means of these operators we are able to reconstruct signals (functions) which are not necessarily continuous. Moreover, our generalization allows us to take the measurement error into account. The goal of this paper is to estimate the rate of approximation by the above operators via high-order modulus of smoothness.

Keywords: Generalized Kantorovich-type sampling operators; L^p spaces; Operator norms; Modulus of smoothness; Order of approximation; Singular integral of Fejér's type; Bandlimited kernels

1. Introduction

The theory of generalized sampling operators was initially developed at RWTH Aachen by P.L. Butzer and his students in the late 1970s and has been extended thoroughly by many authors since then (see e.g. [6,7,19]). A generalized sampling operator generated by a kernel function

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$\varphi \in L^1(\mathbb{R})$ is defined for a uniformly continuous and bounded function $f \in C(\mathbb{R})$ by

$$S_W^{\varphi} f(t) := \sum_{k=-\infty}^{\infty} f\left(\frac{k}{W}\right) \varphi(Wt - k) \quad (t \in \mathbb{R};\ W > 0). \tag{1}$$

The operator is well-defined when the following conditions are satisfied:

$$\sum_{k=-\infty}^{\infty} |\varphi(u-k)| < \infty \quad (u \in \mathbb{R}), \tag{2}$$

the absolute convergence being uniform on compact subsets of $\mathbb{R}$, and

$$\sum_{k=-\infty}^{\infty} \varphi(u-k) = 1 \quad (u \in \mathbb{R}). \tag{3}$$

In [1] the authors introduced the Kantorovich version of operator (1) by replacing the exact value $f(k/W)$ with the Steklov mean of f on the interval $[k/W, (k+1)/W]$,

$$\overline{f}\left(\frac{k}{W}\right) := W \int_{k/W}^{(k+1)/W} f(u)\,du.$$

Such mean values were previously used by L.V. Kantorovich in order to extend the Bernstein polynomials to the space $L^p(\mathbb{R})$, hence the name. The non-linear version of Kantorovich-type sampling operators was considered in [23]. Moreover, various results on Kantorovich-type sampling operators were recently obtained. For more information see e.g. [3,9–11,24,8].

In this paper we generalize the notion of Kantorovich-type sampling operator given in [1] by replacing the Steklov mean with its more general analogue, the Fejér-type singular integral (see Section 2) and get for $f \in L^p(\mathbb{R})$ $(1 \leqslant p \leqslant \infty)$ the operator $(t \in \mathbb{R};\ W > 0;\ n \in \mathbb{N})$

$$S_{W,n}^{\chi,\varphi} f(t) := \sum_{k=-\infty}^{\infty} \left(\int_{-\infty}^{\infty} f(u) nW \chi\left(nW\left(\frac{k}{W} - u\right)\right) du \right) \varphi(Wt - k). \tag{4}$$

The operator is well-defined when the kernel $\varphi \in L^1(\mathbb{R})$ satisfies the conditions (2), (3) and the kernel $\chi \in L^1(\mathbb{R})$ satisfies

$$\int_{-\infty}^{\infty} \chi(u)\,du = 1.$$

Note that the choice of parameter n in the singular integral allows us to vary the length of kernel support which would be otherwise strictly determined by the spacing parameter W.

Let us mention that the operators similar to (4) were introduced in the paper [4], in which a Durrmeyer-type sampling operator is studied with respect to its asymptotic behaviour.

The Kantorovich-type sampling operators represent natural extension of the generalized sampling series to the setting of L^p spaces, an important instance when the functions (signals) are not necessarily continuous. A reason for considering the generalization (4) lies in the fact that measurements in applications are usually done according to some distribution, so the integral component of the operator is already implemented in hardware. Knowing the parameters of this distribution and consequently the corresponding kernel χ of the singular integral, we can choose the appropriate kernel φ to get the desired accuracy of approximation.