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Haar functions in weighted Besov and Triebel–Lizorkin spaces

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Abstract

The paper deals with Haar wavelet bases in function spaces of Besov and Triebel–Lizorkin type with local Muckenhoupt weights. We show that Haar wavelets can be used to characterize such function spaces as far as absolute value of smoothness parameter is small enough and weights fulfill some conditions. The result is based on mapping properties of linear operators involving characteristic functions of dyadic cubes in related spaces and on local means characterization of weighted Besov and Triebel–Lizorkin spaces.

MSC: 46E35; 41A58; 42C40

Keywords: Muckenhoupt weights; Haar wavelets; Besov spaces; Triebel–Lizorkin spaces; Local means

1. Introduction

Wavelet characterizations are widely studied in the context of Besov and Triebel–Lizorkin spaces. We refer to books by Hans Triebel for details and historical remarks, cf. [14,15]. Close relations between local means and the wavelets are described in the unweighted case by H. Triebel in [16]. Using that result H. Triebel proved Haar wavelet characterization in unweighted Besov and Triebel–Lizorkin spaces in [17]. Our aim is to extend his results into function spaces with

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local Muckenhoupt weights $\mathcal{A}_\infty^{loc}$. We obtain the conditions for which Haar wavelets are bases in Besov or Triebel–Lizorkin spaces with local Muckenhoupt weights. We also get the characterization of the above spaces with characteristic functions of dyadic cubes. Local Muckenhoupt weights and corresponding weighted function spaces were introduced by V. Rychkov in 2001, cf. [8]. This class of weights is a generalization of the classical class of Muckenhoupt weights $\mathcal{A}_\infty$. Wavelet characterizations of function spaces with so-called admissible weights were given by Haroske and Triebel in [5]. Later Haroske and Skrzypczak proved the characterization for spaces with Muckenhoupt weights, cf. [3].

We follow the main idea of H. Triebel from [17], that Haar wavelets can serve as kernels of the local means. First we prove that functions from weighted Besov and Triebel–Lizorkin spaces can be represented by characteristic functions of dyadic cubes. Then we introduce local means. To treat Haar wavelets as local means we show embeddings of Besov spaces with local Muckenhoupt weights into class of regular distributions. Then we obtain the main result, Theorem 9.1, using the complex interpolation method given in [11] and a duality argument.

The outline of the paper is as follows. In Section 2 we introduce classes of local Muckenhoupt weights and their main properties. In Section 3 definitions of Besov and Triebel–Lizorkin spaces with local Muckenhoupt weights are formulated. Next, in Section 4, we remind briefly construction of Haar wavelets on $\mathbb{R}^d$. In Section 5 we prove theorems of embeddings of weighted Besov spaces into local L_p spaces. In Section 6 are mentioned some results on dual spaces of spaces with local Muckenhoupt weights. Using results from Section 5 we can treat Haar wavelets as kernels of local means. Definitions and suitable theorems are formulated in Section 7. In Section 8 we give conditions on weights and smoothness parameters of function spaces to characterize such spaces by characteristic functions of dyadic cubes. Finally taking results of last two sections we get the main result, i.e. conditions on which Haar wavelets is a basis in Besov and Triebel–Lizorkin spaces with local Muckenhoupt weights.

2. Classes of weights

Let w be a nonnegative and locally integrable function on $\mathbb{R}^n$. Such functions are called weights and for a measurable set $E w(E)$ denotes $\int_E w(x)\,dx$. We consider $L_p^w(\mathbb{R}^n)$ spaces, i.e., $L_p(\mathbb{R}^n)$ spaces with Lebesgue measure replaced by the measure $w\,dx$.

2.1. Muckenhoupt weights

Let us recall the definition of the Muckenhoupt weights. For more properties see [12, Chapter V].

A weight w belongs to $\mathcal{A}_p$, $w \in \mathcal{A}_p$, $1 < p < \infty$, if

$$A_p(w) := \sup_{Q\subset\mathbb{R}^n} \frac{1}{|Q|^p} \int_Q w(x)\,dx \left(\int_Q w^{1-p'}(x)\,dx\right)^{p-1} < \infty$$

and $w \in \mathcal{A}_1$ if

$$A_1(w) := \sup_{Q\subset\mathbb{R}^n} \frac{w(Q)}{|Q|} \left\| w^{-1} \right\|_{L_\infty(Q)} < \infty$$

where supremum is taken over all cubes $Q \subset \mathbb{R}^n$.

We also put $\mathcal{A}_\infty = \bigcup_{p\geq 1} \mathcal{A}_p$.