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Adaptive thinning of centers for approximation of a large data set by radial functions

Nira Dyn, Pavel Kozlov*

Tel Aviv University, P.O. Box 39040, Tel Aviv, Israel

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Abstract

Given data at a large set of centers, the problem investigated in this article, is how to choose a smaller subset of centers such that the least squares radial function approximation will be with predefined accuracy. Our solution is based on an adaptive thinning strategy, removing in a greedy way less significant centers one by one so as to minimize an anticipated error. The novelty in our approach is the replacement of the anticipated error by simpler "*predicting*" functionals. A predicting functional is a functional which chooses with a high probability the same centers to be removed as an anticipated error. We derive several such predicting functionals for specific radial functions which have the "*functionals consistency*" property, namely that there are functionals which change their values on translates of the radial function in a co-monotone way. Our numerical tests demonstrate good performance of the proposed predicting functionals on piecewise continuous functions.

Keywords: Radial function; Thinning; Anticipated error; Ordering functional; Predicting functional; Functionals consistency

1. Introduction

An ubiquitous problem is to approximate a function from a large set of its samples. Radial function approximation is a well known method used to produce approximants of multivariate

* Corresponding author.
E-mail addresses: niradyn@post.tau.ac.il (N. Dyn), pavkoz@gmail.com (P. Kozlov).

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functions. Time and computer resources for the computation of such an approximant decrease with the decrease in the number of centers involved, while the error between the approximant and the function increases. Also the complexity of the evaluation of such an approximant increases with the number of centers.

Interpolation is a main method for approximation by radial functions [1,10]. Since interpolation uses all the centers, interpolation becomes costly for large data sets. In problems where the approximant may deviate from the data up to a given error, least squares approximation can be employed. In this case a method for choosing a "good" small subset of centers is needed.

Here we develop several thinning methods to reduce the number of centers as much as possible and to obtain an approximant with a predefined accuracy. We use the Adaptive Thinning strategy, removing less significant centers from the initial set one by one in a greedy way. To find such less significant centers we use an *ordering* functional which order centers by their significance. Ordering functionals which were used up to now are different anticipated error functionals estimating the approximation error incurred by the removal of a center. Our main idea is to replace these anticipated error functionals by easy to compute *predicting* functionals, which determine with high probability the same ordering as anticipated error functionals. To develop several such predicting functionals we observe that strictly monotone radial functions have a property which we term *functionals consistency* property, namely there exists a pair of functionals which change their values on any sequence of translates of the radial function in a co-monotone way. Therefore, the values of such functionals determine the same ordering on any sequence of centers. With this approach the thinning algorithm becomes less heavy in computing time when an anticipated error functional is replaced by a simpler predicting functional.

The outline of this article is as follows: in Section 2 we introduce an Adaptive Thinning algorithm which determines an approximant with a predefined accuracy. In Section 3 we propose new predicting functionals for choosing a small set of significant centers. The mathematical reasoning for the choice of the predicting functionals is given in Section 4. In Section 4.1 we present a simplified predicting functional as the first ingredient of our reasoning. We prove a Functionals Consistency Theorem in Section 4.2 and give a probabilistic measure of quality of the predicting functionals in Section 4.4. Concrete predicting functionals are derived in Sections 4.2 and 4.3. In Section 5 we demonstrate the performance of our proposed methods on several examples.

2. Adaptive thinning for least—squares approximation by radial functions

The thinning methods proposed in this article are inspired by [2,3]. The data in [3] is similar to the data considered in the present paper, and also there a subset Y of the given data points is required. But the approximant there is the interpolant to the given function's values at the points of Y, which is piecewise linear on the Delaunay triangulation of Y, while here it is the least squares approximation to the function from the space of shifts of a radial function to the points of Y. Thus we obtain a smooth approximant in the L_2-norm, while the ones in [3] are only continuous and aim at reducing the sup-norm. In [2] a univariate problem, similar to the bivariate one in [3] is considered, and the sets Y obtained for piecewise linear interpolants are also used successfully for approximation by shifts of a radial function. In the present paper we look for thinning methods devised for bivariate least squares approximation.

First we introduce some notation. Consider a set of centers (points) $\Xi \subset D$ and a subset of it $Y_J := \{y_i : i = 1, \dots, J\}$, where D is a bounded domain of $\mathbb{R}^d$, $d \geq 1$. For a function