



Available online at www.sciencedirect.com

ScienceDirect

Journal of Approximation Theory 197 (2015) 62–68

JOURNAL OF
**Approximation
Theory**

www.elsevier.com/locate/jat

Full length article

Basis partition polynomials, overpartitions and the Rogers–Ramanujan identities

George E. Andrews

Department of Mathematics, The Pennsylvania State University, University Park, PA 16802, United States

Received 18 October 2013; received in revised form 29 March 2014; accepted 10 May 2014

Available online 20 May 2014

Communicated by Special Issue Guest Editor

Dedicated with admiration to my friend, Richard Askey

Abstract

In this paper, a common generalization of the Rogers–Ramanujan series and the generating function for basis partitions is studied. This leads naturally to a sequence of polynomials, called BsP-polynomials. In turn, the BsP-polynomials provide simultaneously a proof of the Rogers–Ramanujan identities and a new, more rapidly converging series expansion for the basis partition generating function. Finally the basis partitions are identified with a natural set of overpartitions.

© 2014 Elsevier Inc. All rights reserved.

Keywords: Basis partitions; Overpartitions; Rogers–Ramanujan identities

1. Introduction

The late Hansraj Gupta [8] introduced the concept of basis partitions. Basis partitions are defined in terms of successive ranks [6] or the “rank vector” of a partition.

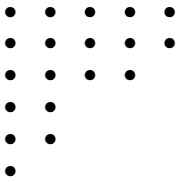
Namely, each partition, π , of a positive integer contains a largest square of nodes in its Ferrers graph. This square is called the Durfee square. If the Durfee square has side d , we define the i th rank r_i of π ($1 \leq i \leq d$) as the difference between the number of nodes in the i th row of the Ferrers graph of π and the number in the i th column. The rank vector for π is (r_1, r_2, \dots, r_d) .

E-mail addresses: gea1@psu.edu, andrews@math.psu.edu.

<http://dx.doi.org/10.1016/j.jat.2014.05.008>

0021-9045/© 2014 Elsevier Inc. All rights reserved.

For example, if π is the partition $5 + 5 + 4 + 2 + 2 + 1$, then its Ferrers graph is:



Its rank vector is $(-1, 0, 1)$.

Gupta [8] showed that for every rank vector, \vec{r} , there is a smallest integer that has a partition with rank vector \vec{r} , and that partition is unique. This partition is called the basis partition for \vec{r} . We let $B(n)$ denote the number of basis partitions of n .

For example, the basis partition for $(-1, 0, 1)$ is $4 + 4 + 4 + 2 + 1$.

In [11], Nolan, Savage and Wilf showed that

$$\sum_{n=0}^{\infty} B(n)q^n = \sum_{n=0}^{\infty} \frac{q^{n^2}(-q; q)_n}{(q; q)_n}, \tag{1.1}$$

where

$$(A; q)_n = (1 - A)(1 - Aq) \cdots (1 - Aq^{n-1}). \tag{1.2}$$

Hirschhorn [10] gave a new proof of (1.1) and related basis partitions to the Rogers–Ramanujan series from the first Rogers–Ramanujan identity [4, p. 113]:

$$\sum_{n=0}^{\infty} \frac{q^{n^2}}{(q; q)_n} = \frac{1}{(q; q^5)_{\infty}(q^4; q^5)_{\infty}}. \tag{1.3}$$

Our central object is to study

$$G(a, x; q) := \sum_{n=0}^{\infty} \frac{a^n q^{n^2} (x; q)_n}{(q; q)_n}. \tag{1.4}$$

Following the work of Nola, Savage and Wilf [11] and of Hirschhorn [10], Alladi had in 2007 considered $G(1, -zq; q)$ and had interpreted the power of z as representing the signature of a basis partition (namely the number of different parts below the Durfee square); he then studied basis partitions combinatorially [1] with emphasis on the signature.

Notice that if we set $x = -q$ and set $a = 1$ in (1.4), we get the series in (1.1), and if we set $x = 0$ and $a = 1$ we get the series in (1.3). We want to find an identity for $G(a, x; q)$ which both leads directly to the Rogers–Ramanujan identities and also provides a new representation of the series in (1.1).

We shall prove

Theorem 1.

$$G(a, x; q) = \frac{1}{(aq; q)_{\infty}} \left(1 + \sum_{n=1}^{\infty} \frac{(aq; q)_{n-1} (1 - aq^{2n}) (-1)^n q^{n(3n-1)/2} a^n B_n(a, x)}{(q; q)_n} \right) \tag{1.5}$$

Download English Version:

<https://daneshyari.com/en/article/4606961>

Download Persian Version:

<https://daneshyari.com/article/4606961>

[Daneshyari.com](https://daneshyari.com)