Available online at www.sciencedirect.com
ScienceDirect
Journal of Approximation Theory 192 (2015) 132–143
Journal of Approximation Theory
www.elsevier.com/locate/jat

Full length article

On the norm of the hyperinterpolation operator on the unit ball

Heping Wang[a,*], Zexia Huang[b], Chunming Li[a], Lixia Wei[a]

[a] School of Mathematical Sciences, Capital Normal University, Beijing 100048, China
[b] School of Mathematics and Computer Engineering, Sihua University, Chengdu 610039, China

Received 22 August 2014; received in revised form 26 October 2014; accepted 2 December 2014
Available online 10 December 2014

Communicated by Yuan Xu

Abstract

We obtain the asymptotic order of the operator norm of the hyperinterpolation operator on the unit ball $\mathbb{B}^d$, $d \geq 2$ with respect to the measure $b_{d,\mu}(1- \mid x \mid^2)^{\mu-1/2}dx$, $\mu \geq 0$, where $b_{d,\mu} = \left(\int_{\mathbb{B}^d}(1- \mid x \mid^2)^{\mu-1/2}dx \right)^{-1}$.
© 2014 Elsevier Inc. All rights reserved.

MSC: 65D05; 65D32

Keywords: Hyperinterpolation; Operator norm; Projection; Regularity condition

1. Introduction

Hyperinterpolation of multivariate continuous functions over compact subsets or manifolds was originally introduced by Sloan (see [15]). It is a discretized orthogonal projection on polynomial subspaces, which provides an approximation method more general than the polynomial

* Corresponding author.
E-mail addresses: wanghp@cnu.edu.cn (H. Wang), xhu_hzx@163.com (Z. Huang), shibei0516@163.com (C. Li), huzhuane@126.com (L. Wei).

http://dx.doi.org/10.1016/j.jat.2014.12.004
0021-9045/© 2014 Elsevier Inc. All rights reserved.

interpolation. Though the idea is very general and flexible, the problem in some sense is easier than the multivariate polynomial interpolation. In recent years, hyperinterpolation has attracted much interest, and a great number of interesting results have been obtained (see [1–4,7,11,8,9,5,13–17,20–22]).

This paper is devoted to studying the operator norm of a hyperinterpolation operator on the unit ball $\mathbb{B}^d = \{x \in \mathbb{R}^d \mid |x| \le 1\}$ with respect to the measure $W_\mu(x)dx$, where $W_\mu(x) = b_{d,\mu}(1-|x|^2)^{\mu-1/2}$, $\mu \ge 0$ is the classical Jacobi weight on $\mathbb{B}^d$, $b_{d,\mu} = \left(\int_{\mathbb{B}^d}(1-|x|^2)^{\mu-1/2}dx\right)^{-1}$. We denote by Π_n^d the subspace of polynomials in d variables with total degree $\le n$. Let S_n be the orthogonal projection from $L^2(\mathbb{B}^d, W_\mu(x)dx)$ onto Π_n^d, i.e.,

$$S_n(f)(x) = \int_{\mathbb{B}^d} f(y)E_n(x,y)W_\mu(y)dy, \tag{1.1}$$

where $E_n(x,y)$ is the reproducing kernel for Π_n^d. We note that $E_n(x,y)$ satisfies the following properties:

(1) For any $x, y \in \mathbb{B}^d$, $E_n(x,y) = E_n(y,x)$;
(2) For any fixed $y \in \mathbb{B}^d$, $E_n(\cdot,y) \in \Pi_n^d$;
(3) For any $P \in \Pi_n^d$ and $x \in \mathbb{B}^d$, $P(x) = S_n(P)(x) = \langle P, E_n(\cdot,x)\rangle$,

where $\langle f,g\rangle = \int_{\mathbb{B}^d} f(x)g(x)W_\mu(x)dx$ is the inner product in $L^2(\mathbb{B}^d, W_\mu(x)dx)$.

For $n \ge 1$, we assume that $Q_n(f) := \sum_{\omega\in\Lambda_n}\lambda_{n,\omega}f(\omega)$ is a positive quadrature formula on $\mathbb{B}^d$ which is exact for Π_{2n}^d, i.e., $\lambda_{n,\omega} > 0$ for any $\omega \in \Lambda_n$, and for all $P \in \Pi_{2n}^d$,

$$\int_{\mathbb{B}^d} P(x)W_\mu(x)dx = Q_n(P) = \sum_{\omega\in\Lambda_n}\lambda_{n,\omega}P(\omega), \tag{1.2}$$

where Λ_n is a finite subset of $\mathbb{B}^d$.

The hyperinterpolation operator L_n on $\mathbb{B}^d$ is defined by

$$L_n(f)(x) = \sum_{\omega\in\Lambda_n}\lambda_{n,\omega}f(\omega)E_n(x,\omega), \qquad f \in C(\mathbb{B}^d). \tag{1.3}$$

Since for any $p \in \Pi_n^d$, $p(\cdot)E_n(x,\cdot) \in \Pi_{2n}^d$ for arbitrary fixed $x \in \mathbb{B}^d$, by (1.2) we get

$$\begin{aligned} p(x) = \langle p, E_n(\cdot,x)\rangle &= \int_{\mathbb{B}^d} p(y)E_n(x,y)W_\mu(y)dy \\ &= \sum_{\omega\in\Lambda_n}\lambda_{n,\omega}p(\omega)E_n(x,\omega) = L_n(p)(x). \end{aligned}$$

This implies that the hyperinterpolation operator L_n is a projection onto Π_n^d, i.e., L_n is a bounded linear operator on $C(\mathbb{B}^d)$ satisfying that $L_n^2 = L_n$ and the range of L_n is Π_n^d. For a linear operator L on $C(\mathbb{B}^d)$, the operator norm $\|L\|$ of L is given by

$$\|L\| := \sup\{\|Lf\| \mid f \in C(\mathbb{B}^d),\ \|f\| \le 1\},$$

where $\|\cdot\|$ is the uniform norm.

In [8], Hansen, Atkinson, and Chien investigated the hyperinterpolation operator L_n on $\mathbb{B}^2$ with $\mu = 1/2$ based on a particular positive quadrature formula and obtained that

Download English Version:

https://daneshyari.com/en/article/4607019

Download Persian Version:

https://daneshyari.com/article/4607019

Daneshyari.com