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Journal of Approximation Theory 191 (2015) 46–57

Journal of Approximation Theory

www.elsevier.com/locate/jat

Full length article

Distribution of interpolation points of maximally convergent multipoint Padé approximants

H.-P. Blatt[a,*], R.K. Kovacheva[b]

[a] Katholische Universität Eichstätt-Ingolstadt, Mathematisch-Geographische Fakultät, 85071 Eichstätt, Germany
[b] Bulgarian Academy of Sciences, Institute of Mathematics and Informatics, Acad. Bonchev Str. 8, 1113 Sofia, Bulgaria

Received 23 December 2013; received in revised form 1 April 2014; accepted 10 April 2014
Available online 19 April 2014

Communicated by Guillermo López Lagomasino

Dedicated to the memory of Andrei A. Gonchar and Herbert Stahl

Abstract

The paper investigates the distribution of interpolation points of m_1-maximally convergent multipoint Padé approximants with numerator degree $\leq n$ and denominator degree $\leq m_n$ for meromorphic functions f on a compact set $E \subset \mathbb{C}$, where $m_n = o(n/\log n)$ as $n \to \infty$. It is shown that the normalized counting measures (resp. their associated balayage measures onto the boundary of E) converge for a subsequence in the weak* sense to the equilibrium measure μ_E of E if the multipoint Padé approximants for one single function f converge exactly in m_1-measure on the maximal Green domain of meromorphy $E_{\rho(f)}$.

MSC: 41A20; 41A21; 30E10

Keywords: Padé approximation; m_1-maximal convergence

1. Introduction

Let E be a compact set in $\mathbb{C}$ with connected complement $\Omega = \overline{\mathbb{C}} \setminus E$. The set Ω is called *regular* if there exists a Green function $G(z) = G(z, \infty)$ in Ω with pole at ∞ satisfying

* Corresponding author.
E-mail addresses: hans.blatt@ku-eichstaett.de, hans.blatt@ku.de (H.-P. Blatt), rkovach@math.bas.bg (R.K. Kovacheva).

http://dx.doi.org/10.1016/j.jat.2014.04.004
0021-9045/

$G(z) \to 0$ as $z \to \partial\Omega$. Then

$$\lim_{z\to\infty}(G(z) - \log|z|) = -\log \operatorname{cap} E,$$

where cap E is the logarithmic capacity, and cap $E > 0$ since Ω is regular. We define by

$$E_\rho := \{z \in \Omega : G(z) < \log \rho\} \cup E, \quad \rho > 1$$

the *Green domains* and set $E_1 := \operatorname{int}(E) = E^\circ$. If Ω is regular, then the equilibrium measure μ_E of E exists. We denote the boundary of E_ρ by Γ_ρ.

For $B \subset \mathbb{C}$, we denote by $\overline{B}$ its closure and by ∂B the boundary of B and we use $\|\cdot\|_B$ for the supremum norm on B. Moreover, let $C(B)$ be the class of continuous, complex-valued functions in B, and $\mathcal{A}(B)$ (resp. $\mathcal{M}(B)$) represents the subclass of functions that are holomorphic (resp. meromorphic) in some open neighborhood of B.

Given $n, m \in \mathbb{N}_0$, $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$, let $\mathcal{R}_{n,m}$ be the collection of the rational functions

$$\mathcal{R}_{n,m} := \{r = p/q : p \in \mathcal{P}_n,\ q \in \mathcal{P}_m,\ q \not\equiv 0\}$$

where $\mathcal{P}_n$ (resp. $\mathcal{P}_m$) denotes the set of algebraic polynomials with degree at most n (resp. m).

If $f \in C(E) \cap \mathcal{A}(E^\circ)$ is not an entire function, then there exists a maximal $\rho \geq 1$ such that f has a holomorphic continuation to E_ρ. If $\rho > 1$, then a sequence of polynomials $p_n \in \mathcal{P}_n$, $n \in \mathbb{N}$, is said to *converge maximally* to f on E if

$$\limsup_{n\to\infty} \|f - p_n\|_E^{1/n} = \frac{1}{\rho}. \tag{1.1}$$

(Walsh [17]). Moreover, Walsh proved [17, Corollary on p. 81, section 4.7] that

$$\limsup_{n\to\infty} \|f - p_n\|_{E_\sigma}^{1/n} = \frac{\sigma}{\rho}, \quad 1 < \sigma < \rho, \tag{1.2}$$

for maximally convergent polynomials. Examples for such maximally convergent polynomials are best uniform polynomial approximants to f on E. Another example are interpolating polynomials p_n to f with respect to $\mathcal{P}_n$ for particular choices of point sets

$$Z_n : z_{0,n}, \ldots, z_{n,n} \in E \quad (n \in \mathbb{N}).$$

In the case that these points are not pairwise distinct, we use Hermite interpolation. Given the *normalized counting measure* of Z_n, i.e.,

$$\tau_n(B) := \frac{\#\{z \in Z_n : z \in B\}}{n+1},$$

it is well known that p_n will converge maximally to f on E if the measures τ_n converge in the weak* sense to the equilibrium measure μ_E of E (Walsh [17]).

Conversely, Grothmann [5] obtained the following result:

If $f \in \mathcal{A}(E)$ is not entire and $\{p_n\}_{n\in\mathbb{N}}$ is a sequence of maximally convergent polynomials interpolating on the sets $\{Z_n\}_{n\in\mathbb{N}}$, $Z_n \subset E$, then there exists a subset $\Lambda \subset \mathbb{N}$ such that

$$\widehat{\tau_n} \xrightarrow[n\in\Lambda, n\to\infty]{*} \mu_E,$$

where $\widehat{\tau_n}$ is the balayage of τ_n onto the boundary of E.