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Simultaneous approximation of a multivariate function and its derivatives by multilinear splines

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Abstract

In this paper we consider the approximation of a function by its interpolating multilinear spline and the approximation of its derivatives by the derivatives of the corresponding spline. We obtain the exact uniform approximation error on classes of functions with moduli of continuity bounded above by certain majorants.
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1. Basic definitions and notation

Let $\mathbf{x} = (x_1, x_2, \dots, x_n)$ be a point in Euclidean space $\mathbb{R}^n$. By C_D we denote the class of functions $f(\mathbf{x}) = f(x_1, x_2, \dots, x_n)$ that are continuous on the domain $D := [0, 1]^n \subset \mathbb{R}^n$.

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We consider a vector $\mathbf{r} \in \{0, 1\}^n$, i.e. a vector having n components each being either 0 or 1. Let $C_D^{\mathbf{r}}$ be the class of functions, $f(\mathbf{x}) \in C_D$, with continuous derivatives

$$f^{(\mathbf{t})}(\mathbf{x}) = \frac{\partial^{\sum_{i=1}^n t_i} f}{\partial x^{t_1} \cdots \partial x^{t_n}}(\mathbf{x}),$$

where $\mathbf{t} \in \{0, 1\}^n$ and $t_i \le r_i$ for each $i = 1, \ldots, n$. We define $f^{(\mathbf{0})}(\mathbf{x}) := f(\mathbf{x})$ and $C_D^{\mathbf{0}} := C_D$.

For any function $f(\mathbf{x}) \in C_D$, consistent with literature, we denote the uniform norm as

$$\|f\|_C := \max\{|f(\mathbf{x})| : \mathbf{x} \in D\}.$$

The next two definitions introduce two types of moduli of continuity of a given function f, both characterizing the smoothness of the original function f.

Definition 1. If the function $f(\mathbf{x})$ is bounded for $x_i \in [a_i, b_i], i = 1, \ldots, n$, then its total modulus of continuity, $\omega(f; \boldsymbol{\tau})$, is defined as follows

$$\begin{aligned}\omega(f; \boldsymbol{\tau}) &:= \omega(f; \mathbf{a}, \mathbf{b}; \boldsymbol{\tau}) \\ &= \sup\{|f(\mathbf{x}) - f(\mathbf{y})| : |x_i - y_i| \le \tau_i; x_i, y_i \in [a_i, b_i]\},\end{aligned}$$

where $0 \le \tau_i \le b_i - a_i$, for $i = 1, \ldots, n$ and $\boldsymbol{\tau} := (\tau_1, \ldots, \tau_n)$, $\mathbf{a} := (a_1, \ldots, a_n)$, $\mathbf{b} := (b_1, \ldots, b_n)$.

In addition, we consider the following l_p distances, $1 \le p < \infty$, between points $\mathbf{x}, \mathbf{y} \in D \subset \mathbb{R}^n$,

$$\|\mathbf{x} - \mathbf{y}\|_p := \sqrt[p]{\sum_{i=1}^n |x_i - y_i|^p}.$$

Definition 2. For the function $f(\mathbf{x}) \in C_D$ and for given p, $1 \le p < \infty$, we define the modulus of continuity of function f with respect to p to be

$$\omega_p(f; \gamma) := \sup\left\{|f(\mathbf{x}) - f(\mathbf{y})| : \mathbf{x}, \mathbf{y} \in D, \|\mathbf{x} - \mathbf{y}\|_p \le \gamma\right\}, \quad 0 \le \gamma \le d_p,$$

where $d_p := \max\{\|\mathbf{x} - \mathbf{y}\|_p : \mathbf{x}, \mathbf{y} \in D \subset \mathbb{R}^n\}$.

We point out that

$$d_p = \sqrt[p]{n}.$$

Note that the moduli of continuity of all suitable functions have some common properties. We call all functions (univariate or multivariate) with these properties *functions of the moduli of continuity type* and use them to define classes of smoothness of functions.

Definition 3. Function $\Omega(\boldsymbol{\tau})$ is called a function of modulus of continuity type, or MC-type function (for short), if the following properties hold for any vectors $\boldsymbol{\gamma}, \boldsymbol{\tau} \in \mathbb{R}_+^n := \{\mathbf{x} \in \mathbb{R}^n : x_i \ge 0, i = 1, \ldots, n\}$:

1. $\Omega(\mathbf{0}) = 0$.
2. $\Omega(\boldsymbol{\tau}) := \Omega(\tau_1, \ldots, \tau_n)$ is non-decreasing (in each coordinate).
3. $\Omega(\boldsymbol{\tau} + \boldsymbol{\gamma}) \le \Omega(\boldsymbol{\tau}) + \Omega(\boldsymbol{\gamma})$, that is $\Omega(\boldsymbol{\tau})$ is subadditive.
4. $\Omega(\boldsymbol{\tau})$ is continuous for all $\tau_i, i = 1, \ldots, n$.

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