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Interpolation on cycloidal spaces

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Abstract

Hermite interpolation problems on cycloidal spaces are analyzed. Newton and Aitken–Neville formulae are obtained. Numerical examples are included.
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1. Introduction

The cycloidal space of dimension $n+1$ is defined for $n \geq 2$ by

$$C_n := \langle \cos x, \sin x, 1, x, x^2, \ldots, x^{n-2} \rangle$$

and for $n = 1$, $C_1 := \langle \cos x, \sin x \rangle$. Cycloidal spaces have been used in geometric design to deal with shape preserving representations that allow circles to be represented by their natural parameterization. They also allow the designer to deal with important curves in civil engineering, such as cycloids and helices. In [9], an extension of cubic curves was considered by using curves whose components are functions in the cycloidal space C_3. In [5], curves in C_4 have been considered. In [2], the cycloidal space C_5 was analyzed and some properties of the general cycloidal spaces C_n were discussed. Shape preserving representations in general spaces of the form $\langle u(x), v(x), 1, x, x^2, \ldots, x^{n-2} \rangle$ with suitable functions u and v have been considered in [4].

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In this paper we focus on interpolation problems on cycloidal spaces. Mühlbach derived general interpolation formulae in extended complete Chebyshev spaces (see [6,7]), also discussed in [8]. We shall deduce some interpolation formulae that closely resemble those derived by Mühlbach. Since cycloidal spaces are not extended Chebyshev spaces on intervals of arbitrary length (see Section 4), formulae in [6,7] need to be extended under weaker hypotheses. We also take advantage of the invariance under translations of cycloidal spaces and the fact that C_n contains as a subspace P_{n-2}, the space of polynomials of degree not greater than $n-2$.

In Section 2, we consider the Hermite interpolation problem on C_n and derive a Newton formula for the cycloidal interpolant. We describe recurrence relations for cycloidal divided differences and an Aitken–Neville formula. We also suggest an algorithm for implementing Newton's formula. In Section 3, we relate the cycloidal interpolant of a function f with polynomial interpolants of the functions f, cosine and sine and obtain a bound for the interpolation error. We also illustrate with some examples the results previously discussed. They confirm the ability of cycloidal interpolants to reproduce oscillations. They also illustrate the fact that a cycloidal interpolant can have much lower interpolation errors outperforming polynomial interpolation. Finally, in Section 4 we relate the results in this paper with Mühlbach's results through the critical lengths of the spaces introduced in [2].

2. The Hermite interpolation problem on cycloidal spaces

An extended sequence of nodes is a sequence of points $x_0, \ldots, x_n$, not necessarily distinct. We pose the *Hermite interpolation problem* on C_n at an extended sequence of nodes $x_0, \ldots, x_n$: find $c \in C_n$ such that $\lambda_i c = \lambda_i f,\ i = 0, \ldots, n$, where

$$\lambda_i f := f^{(r_i-1)}(x_i), \quad r_i = \#\{j \le i \mid x_j = x_i\}. \tag{1}$$

If $x_0, \ldots, x_n$ are nodes such that, there exists a unique solution of the Hermite interpolation problem for any f, the unique solution will be called the *cycloidal interpolant* of f and will be denoted by $C(f; x_0, \ldots, x_n)$. The cycloidal interpolant is invariant under translations as shown in the following result.

Proposition 1. *Assume that the Hermite interpolation problem at $x_0, \ldots, x_n$ has a unique solution in C_n, $n \ge 1$. Then for any $t \in \mathbf{R}$, the Hermite interpolation problem at $x_0+t, \ldots, x_n+t$ has also a unique solution in C_n,*

$$C(f; x_0+t, \ldots, x_n+t)(x) = C(f(\cdot + t); x_0, \ldots, x_n)(x-t). \tag{2}$$

Proof. Let us show that the interpolant at $x_0+t, \ldots, x_n+t$ exists for any function f. Let $c(x) := C(f(\cdot+t); x_0, \ldots, x_n)(x) \in C_n$. The space C_n, $n \ge 1$, is invariant under translations in the sense that, if $c \in C_n$, then $c_t(x) := c(x-t)$ belongs to C_n for any $t \in \mathbf{R}$. Thus we have that $c_t \in C_n$ interpolates f at $x_0+t, \ldots, x_n+t$ because $c_t^{(r_i-1)}(x_i+t) = c^{(r_i-1)}(x_i) = f^{(r_i-1)}(x_i+t)$, $r_i = \#\{j \le i \mid x_j = x_i\}$, $i = 0, \ldots, n$. Then the existence, uniqueness and (2) follows. □

An alternative basis to $(\cos x, \sin x, 1, x, \ldots, x^{n-2})$ for C_n is given by

$$\varphi_0(x) := \cos x, \qquad \varphi_i(x) := \int_0^x \varphi_{i-1}(y)dy, \quad i = 1, \ldots, n. \tag{3}$$

Clearly $\varphi_i \in C_i$, for all $i = 0, \ldots, n$, and the linear independence follows from the fact that $\varphi_i(0) = \varphi_i'(0) = \cdots = \varphi_i^{(i-1)}(0) = 0$, $\varphi_i^{(i)}(0) = 1$. The function φ_n is the fundamental solution

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