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Exceptional Meixner and Laguerre orthogonal polynomials☆

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Abstract

Using Casorati determinants of Meixner polynomials $(m_n^{a,c})_n$, we construct for each pair $\mathcal{F} = (F_1, F_2)$ of finite sets of positive integers a sequence of polynomials $m_n^{a,c;\mathcal{F}}$, $n \in \sigma_{\mathcal{F}}$, which are eigenfunctions of a second order difference operator, where $\sigma_{\mathcal{F}}$ is certain infinite set of nonnegative integers, $\sigma_{\mathcal{F}} \subsetneq \mathbb{N}$. When c and $\mathcal{F}$ satisfy a suitable admissibility condition, we prove that the polynomials $m_n^{a,c;\mathcal{F}}$, $n \in \sigma_{\mathcal{F}}$, are actually exceptional Meixner polynomials; that is, in addition, they are orthogonal and complete with respect to a positive measure. By passing to the limit, we transform the Casorati determinant of Meixner polynomials into a Wronskian type determinant of Laguerre polynomials $(L_n^\alpha)_n$. Under the admissibility conditions for $\mathcal{F}$ and α, these Wronskian type determinants turn out to be exceptional Laguerre polynomials.
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1. Introduction

In [12], we have introduced a systematic way of constructing exceptional discrete orthogonal polynomials using the concept of dual families of polynomials. Using Charlier polynomials, we

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applied this procedure to construct exceptional Charlier polynomials and, passing to the limit, exceptional Hermite polynomials. The purpose of this paper is to extend this construction using Meixner and Laguerre polynomials.

Exceptional orthogonal polynomials $p_n, n \in X \subsetneq \mathbb{N}$, are complete orthogonal polynomial systems with respect to a positive measure which in addition are eigenfunctions of a second order differential operator. They extend the classical families of Hermite, Laguerre and Jacobi. The last few years have seen a great deal of activity in the area of exceptional orthogonal polynomials (see, for instance, [7,16,17] (where the adjective exceptional for this topic was introduced), [18,19,21–23,28,30,31,33,37,38,40] and the references therein).

In the same way, exceptional discrete orthogonal polynomials are complete orthogonal polynomial systems with respect to a positive measure which in addition are eigenfunctions of a second order difference operator, extending the discrete classical families of Charlier, Meixner, Krawtchouk and Hahn, or Wilson, Racah, etc., if orthogonal discrete polynomials on nonuniform lattices are considered [12,31,41]. One can also add to the list the exceptional q-orthogonal polynomials related to second order q-difference operators [31,32,34–36].

The most apparent difference between classical or classical discrete orthogonal polynomials and their exceptional counterparts is that the exceptional families have gaps in their degrees, in the sense that not all degrees are present in the sequence of polynomials (as it happens with the classical families) although they form a complete orthonormal set of the underlying L^2 space defined by the orthogonalizing positive measure. This means in particular that they are not covered by the hypotheses of Bochner's and Lancaster's classification theorems (see [4] or [26]) for classical and classical discrete orthogonal polynomials, respectively. Exceptional orthogonal polynomials have been applied to shape-invariant potentials [37], supersymmetric transformations [18], to discrete quantum mechanics [31], mass-dependent potentials [28], and to quasi-exact solvability [40].

As mentioned above, we use the concept of dual families of polynomials to construct exceptional discrete orthogonal polynomials (see [27]). One can then also construct examples of exceptional orthogonal polynomials by taking limits in some of the parameters in the same way as one goes from classical discrete polynomials to classical polynomials in the Askey tableau.

Definition 1.1. Given two sets of nonnegative integers $U, V \subset \mathbb{N}$, we say that the two sequences of polynomials $(p_u)_{u\in U}, (q_v)_{v\in V}$ are dual if there exists a couple of sequences of numbers $(\xi_u)_{u\in U}, (\zeta_v)_{v\in V}$ such that

$$\xi_u p_u(v) = \zeta_v q_v(u), \quad u \in U, v \in V. \tag{1.1}$$

Duality has shown to be a fruitful concept regarding discrete orthogonal polynomials, and its utility will be again manifested in the exceptional discrete polynomials world. Indeed, as we pointed out in [12], it turns out that duality interchanges exceptional discrete orthogonal polynomials with the so-called Krall discrete orthogonal polynomials. A Krall discrete orthogonal family is a sequence of polynomials $(p_n)_{n\in\mathbb{N}}$, p_n of degree n, orthogonal with respect to a positive measure which, in addition, are also eigenfunctions of a higher order difference operator. A huge amount of families of Krall discrete orthogonal polynomials have been recently introduced by the author by means of a certain Christoffel transform of the classical discrete measures of Charlier, Meixner, Krawtchouk and Hahn (see [8,9,13]). A Christoffel transform is a transformation which consists in multiplying a measure μ by a polynomial r. It has a long tradition in the context of orthogonal polynomials: it goes back a century and a half ago when E.B. Christoffel (see [6] and also [39]) studied it for the particular case $r(x) = x$.

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