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Journal of Approximation Theory 173 (2013) 122–157
Journal of Approximation Theory
www.elsevier.com/locate/jat

Full length article

Direct and inverse approximation theorems for the p-version of the finite element method in the framework of weighted Besov spaces, part III: Inverse approximation theorems

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Received 13 November 2011; received in revised form 24 December 2012; accepted 3 May 2013
Available online 28 May 2013

Communicated by H. Mhaskar

Abstract

This is the third of a series devoted to the direct and inverse approximation theorems of the p-version of the finite element method in the framework of the Jacobi-weighted Besov and Sobolev spaces. In this paper we derive the inverse algebraic approximation in terms of the Jacobi-weighted Besov spaces and prove the inverse theorems for the finite element solutions of the p-version in the Chebyshev-weighted Besov spaces based upon the convergence rate measured in the energy norms.
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Keywords: Jacobi-weighted Besov spaces; Jacobi-weighted Sobolev spaces; Jacobi weights; Algebraic approximation; The p-version of finite element method; Optimal rate of convergence; Direct and inverse approximation theorem

1. Introduction

This paper is the third of a series devoted to the direct and inverse approximation theorems of the p-version of the finite element method (FEM) and will focus on the inverse approximation.

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0021-9045/$ - see front matter © 2013 Elsevier Inc. All rights reserved.
http://dx.doi.org/10.1016/j.jat.2013.05.001

The first and second parts of the series [5,6] concentrate on the direct approximation for singular and smooth solutions and were published a decade ago. The direct approximation theorems give the convergence rate of the finite element approximation to problems for which the solutions belong to certain functional spaces, and the inverse approximation theorems characterize with certain functional spaces the regularity of solutions for problems under consideration based upon asymptotical convergence rate of the finite element approximation. Although numerous papers on a priori error estimation of the p-version of the finite element method have been published, there are only a few papers on the inverse approximation for algebraic polynomials in the literature e.g. see [7,14,31,35], but in our knowledge, no paper on inverse approximation of the finite element (FE) solutions of the p-version is available.

The inverse approximation for the FE solutions of the p-version has never been addressed and no progress has been made on this important part of the approximation theory of the FEM since the p-version of FEM was introduced in later 1970s. There are two substantial difficulties which severely affect the research on this subject, we would elaborate them here in a detailed way.

At first, it is not clear which functional spaces are appropriate for the inverse approximation for algebraic polynomials, which is an issue of the polynomial approximation, but it has significant impact on the approximation theory of the p-version of the FEM. It has been realized that the periodic Sobolev spaces H^k are appropriate for the approximation by trigonometric polynomials [13,31] because the following two inequalities are valid for the trigonometric polynomials in the periodic Sobolev spaces.

$$\inf_{\phi\in\mathcal{T}_n(I_\pi)} \|u-\phi\|_{H^l(I_\pi)} \le C(n+1)^{-(k-l)}\|u\|_{H^k(I_\pi)}, \quad \forall u\in H^k(I_\pi),\ k\ge l\ge 0 \tag{1.1}$$

where $\mathcal{T}_n(I_\pi)$ is a set consisting of all trigonometric polynomial of degree $n\ge 0$ on $I_\pi = (-\pi,\pi)$, and

$$|\phi|_{H^k(I_\pi)} \le C(n+1)^{k-l}|\phi|_{H^l(I_\pi)}, \quad \forall \phi\in\mathcal{T}_n(I),\ k\ge l\ge 0. \tag{1.2}$$

They are called the Jackson inequality and the Bernstein inequality in the literature, respectively. With the help of the above two inequalities and the argument for Theorem 3.4, one can prove that $u\in B^{s+l}(I_\pi)$ if there exists $\phi\in\mathcal{T}_n(I_\pi)$ for any integer $n\ge 0$ such that

$$\|u-\phi\|_{H^l(I_\pi)} \le C(n+1)^{-s}$$

where $B^s(I_\pi) = (H^l(I_\pi), H^k(I_\pi))_{\theta,\infty}$ with $\theta = \frac{s-l}{k-l}\in(0,1)$ and integers $k>l\ge 0$ is the Besov space which is an interpolation space associated with the periodic Sobolev spaces. Actually one can generalize the Jackson inequality for functions $u\in B^s(I_\pi)$ with $s>0$, i.e., there exists $\phi\in\mathcal{T}_n(I_\pi)$ such that

$$\|u-\phi\|_{H^l(I_\pi)} \le C(n+1)^{-(s-l)}\|u\|_{B^s(I_\pi)}, \quad 0\le l<s. \tag{1.3}$$

Thus a combination of (1.2) and (1.3) forms a perfect pair of direct and inverse approximation theorems for trigonometric polynomials.

Unfortunately, the Bernstein inequality (1.2) is not valid for algebraic polynomials, i.e., for any algebraic polynomial ϕ of degree $n\ge 0$

$$|\phi|_{H^k(I)} \le C(n+1)^{2(k-l)}|\phi|_{H^l(I)}, \quad 0\le l\le k, \tag{1.4}$$

where $I=(-1,1)$, and it is indicated in [2] that inverse inequality (1.4) is sharp and cannot be improved. Adopting the argument of Theorem 3.4, one can prove that $u\in B^{l+\frac{s}{2}}(I)$ only, instead

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