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Bounding the Lebesgue constant for Berrut's rational interpolant at general nodes

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Abstract

It has recently been shown that the Lebesgue constant for Berrut's rational interpolant at equidistant nodes grows logarithmically in the number of interpolation nodes. In this paper we show that the same holds for a very general class of well-spaced nodes and essentially any distribution of nodes that satisfies a certain regularity condition, including Chebyshev–Gauss–Lobatto nodes as well as extended Chebyshev nodes.

Keywords: Barycentric rational interpolation; Lebesgue constant

1. Introduction

For $n \in \mathbb{N}$, let $X_n = \{x_0, x_1, \ldots, x_n\}$ be a set of $n + 1$ distinct *interpolation nodes* in the real interval $[a, b]$, and let $B_n = \{b_0, b_1, \ldots, b_n\}$ be a corresponding set of *cardinal basis functions*, that is, $b_j(x_k) = \delta_{jk}$. The norm of the linear interpolation operator which maps $f \in \mathcal{C}^0[a, b]$ to

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$g = \sum_{j=0}^{n} b_j f(x_j)$ is also known as the *Lebesgue constant* [8,11,12]

$$\Lambda(X_n, B_n) = \max_{a \le x \le b} \sum_{j=0}^{n} \left| b_j(x) \right|,$$

and one is interested in choices of nodes X_n and basis functions B_n such that $\Lambda(X_n, B_n)$ is small, as this leads to small bounds on the approximation error $\|f - g\|_\infty$, measured in the maximum norm.

In the case of polynomial interpolation, when B_n is the set of Lagrange basis polynomials with respect to X_n, it is well known that the Lebesgue constant grows logarithmically with n for Chebyshev nodes, but exponentially for equidistant nodes [6]. However, much smaller Lebesgue constants are observed for rational interpolation at equidistant nodes [2,3,7,13]. In particular, Bos et al. [4,5] show that the Lebesgue constant for Berrut's first rational interpolant [1] with basis functions

$$b_i(x) = \frac{(-1)^i}{x - x_i} \Big/ \sum_{j=0}^{n} \frac{(-1)^j}{x - x_j}, \quad i = 0, \ldots, n,$$

grows logarithmically in the number of equidistant interpolation nodes, and that the same holds for Berrut's second rational interpolant [1] and the general family of barycentric rational interpolants which was introduced by Floater and Hormann [9]. Moreover, Hormann et al. [10] show that this behaviour extends to *quasi-equidistant* nodes with a bounded global mesh ratio.

In this work we show that for essentially *any* reasonable choice of interpolation nodes X_n, the Lebesgue constant for Berrut's first rational interpolant,

$$\Lambda(X_n) = \max_{a \le x \le b} \frac{\sum_{j=0}^{n} \frac{1}{|x - x_j|}}{\left| \sum_{j=0}^{n} \frac{(-1)^j}{x - x_j} \right|}, \tag{1}$$

is bounded from above by $c \ln n$ for some constant $c > 0$. More precisely, we derive this result for *well-spaced* nodes (Section 2), and show that such nodes can be generated by a distribution function satisfying a regularity condition (Section 3). We emphasize that our main goal is to prove the logarithmic growth, and that we are less interested in deriving either tight bounds or exact constants c. We conclude the paper by discussing several examples (Section 4).

1.1. Preliminaries

Without loss of generality we assume that the nodes in X_n are ordered,

$$x_0 < x_1 < \cdots < x_n,$$

and that $x_0 = 0$ and $x_n = 1$, because the Lebesgue constant in (1) is invariant with respect to linear transformations of the interpolation nodes. We further denote the length of the k-th *subinterval* by

$$h_k = x_{k+1} - x_k, \quad k = 0, \ldots, n-1 \tag{2}$$

and restrict our discussion to the interpolation interval $[a, b] = [x_0, x_n] = [0, 1]$.