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Nonuniform support recovery from noisy random measurements by Orthogonal Matching Pursuit[☆]

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Abstract

This article considers nonuniform support recovery via Orthogonal Matching Pursuit (OMP) from noisy random measurements. Given m admissible random measurements (of which Subgaussian measurements is a special case) of a fixed s-sparse signal x in $\mathbb{R}^n$ corrupted with additive noise, we show that under a condition on the minimum magnitude of the nonzero components of x, OMP can recover the support of x exactly after s iterations with overwhelming probability provided that $m = O(s \log n)$. This extends the results of Tropp and Gilbert (2007) [53] to the case with noise. It is a real improvement over previous results in the noisy case, which are based on mutual incoherence property or restricted isometry property analysis and require $O(s^2 \log n)$ random measurements. In addition, this article also considers sparse recovery from noisy random frequency measurements via OMP. Similar results can be obtained for the partial random Fourier matrix via OMP provided that $m = O(s(s + \log(n - s)))$. Thus, for some special cases, this answers the open question raised by Kunis and Rauhut (2008) [34], and Tropp and Gilbert (2007) [53].
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Keywords: Orthogonal Matching Pursuit; Compressed sensing; Nonuniform sparse recovery; Support recovery

1. Introduction

Compressed sensing is a new type of sampling theory, that predicts sparse signal can be reconstructed from what was previously believed to be incomplete information [10,11,18]. In

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compressed sensing, one considers the following model:

$$y = Ax + z, \tag{1.1}$$

where A is a known $m \times n$ measurement matrix (with $m \ll n$) and $z \in \mathbb{R}^m$ is a vector of measurement errors. The goal is to reconstruct the unknown signal $x \in \mathbb{R}^n$ based on y and A. Clearly, in general this task is impossible since even if A has full rank then there are infinitely many solutions to this equation. The situation dramatically changes if x is sparse, i.e., $\|x\|_0 = |\{j : x_j \neq 0\}|$ is small.

The approach for solving this problem probably comes first to mind is to search for the sparsest vector in the feasible set of possible solutions, which leads to the l_0-minimization. However, solving l_0-minimization directly is NP-hard in general and thus is computationally infeasible [35,39]. Then it is natural to consider the method of basis pursuit (BP), which can be viewed as a convex relaxation of l_0-minimization that consists in solving the following l_1-minimization problem:

$$\min_{\tilde{x}\in\mathbb{R}^n} \|\tilde{x}\|_1 \quad \text{subject to } A\tilde{x} - y \in \mathcal{B}, \tag{$L_{1,\mathcal{B}}$}$$

where $\mathcal{B}$ is a bounded set determined by noise structure. For instance, $\mathcal{B} = \{0\}$ in the noiseless case and $\mathcal{B} = \{r : \|r\|_2 \leq b\}$ or $\mathcal{B} = \{r : \|A^*r\|_\infty \leq b_\infty\}$ in the noisy case. For any $p \in [1, \infty)$, $u \in \mathbb{R}^d$, denote $\|u\|_p = \left(\sum_{j=1}^d |u_j|^p\right)^{1/p}$ and $\|u\|_\infty = \max_j |u_j|$. l_1-minimization problems with different types of constraints have been well studied in the literature [7–14,18–21, 25,24,33]. Donoho et al. [20] considered constrained l_1-minimization under l_2 constraint. Candès and Tao [14] introduced the Dantzig Selector, which is a constrained l_1-minimization under l_∞ constraint. Now it has been shown that l_1-minimization recovers all s-sparse vectors with small or zero errors provided that the measurement matrix A satisfies a restricted isometry property (RIP) condition $\delta_{cs} \leq C$ for some constants $c, C > 0$ [14,9,12,25,7,24,37]. Let us mention a few results, the condition $\delta_{2s} < 0.414$ was used in Candès [9], $\delta_{2s} < 0.453$ in Foucart and Lai [25], $\delta_{2s} < 0.472$ with the provision that s is either large or a multiple of 4 in Cai et al. [7] and $\delta_{2s} < 0.493$ in Mo and Li [37]. For an $m \times n$ matrix A and $s \leq n$, the RIP constant δ_s [11,13,19] is defined as the smallest number such that for all s-sparse vectors $\tilde{x} \in \mathbb{R}^n$,

$$(1 - \delta_s)\|\tilde{x}\|_2^2 \leq \|A\tilde{x}\|_2^2 \leq (1 + \delta_s)\|\tilde{x}\|_2^2.$$

Note that it is hard to check that a deterministic matrix A has a small RIP constant. So the strategy is to prove that a random matrix satisfies the RIP condition. It is now well known [11,2,36,49] that many types of random measurement matrices such as Gaussian matrices or Subgaussian matrices have RIP constant $\delta_s \leq \delta$ with overwhelming probability provided that

$$m \geq C\delta^{-2} s \log(n/s). \tag{1.2}$$

Up to the constant, the lower bounds for Gelfand widths of l_1-balls [27,26] show that this dependence on n and s is optimal. The RIP condition also holds for a rich class of structured random matrices [11,49,33,48,44,47,43]. The fast multiply partial random Fourier matrix has RIP constant $\delta_s \leq \delta$ with very high probability provided that $m = O(\delta^{-2} s (\log n)^4)$ [11,49,33]. Based on its RIP guarantees, with high probability, BP can recover every s-sparse vector with small or zero errors from $O(s \log(n/s))$ (or with additional log factors in n) random measurements. But in practice, BP is too expensive in large-scale applications. In fact, numerous researchers have

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