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A note on tractability of multivariate analytic problems[☆]

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A B S T R A C T

In this paper we study d-variate approximation for weighted Korobov spaces in the worst-case setting. The considered algorithms use finitely many evaluations of arbitrary linear functionals. We give matching necessary and sufficient conditions for some notions of tractability in terms of two weight parameters. Our result is an affirmative answer to a problem which is left open in a recent paper of Kritzer, Pillichshammer and Woźniakowski.

1. Introduction

Denote by $\mathbb{N}$ the set of all positive integers. Multivariate computational problems are defined on classes of functions depending on $d \in \mathbb{N}$ variables with large or even huge d. Such problems are usually solved by algorithms that use finitely many information operations. In this paper information operation is defined as the evaluation of a linear functional. The information complexity $n(\varepsilon, d)$ is defined as the minimal number of linear functionals which are needed to find the solutions to within an error threshold ε.

Research on tractability of multivariate continuous problems started in 1994 (see [10]). Since then a huge number of results emerged on this topic. Nowadays tractability of multivariate problems is a very active research area (see [7–9] and the references therein). Tractability of multivariate problems has been studied in different settings including the worst case, the probabilistic case and the average

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case setting, but in this paper we limit ourselves to the worst case setting. Roughly speaking, a problem is intractable if the information complexity is an exponential function of ε^{-1} or d. Otherwise, the problem is tractable.

In the past, almost all papers related to tractability were focused on function classes with finite smoothness. However, the case of analytic or infinitely many times differentiable functions is also of interest. Recently, the papers [2,1,3–6] considered tractability for some multivariate analytic function classes. In this paper we will continue the research on tractability of multivariate approximation in $H(K_{d,\mathbf{a},\mathbf{b}})$ (see Section 2 for its definition) which was studied in [1,4,6]. More specifically, P. Kritzer, F. Pillichshammer and H. Woźniakowski [6, Theorem 5.2] assumed that a limit exists and suggested studying the case when this limit does not exist. Furthermore, the authors wondered if strong polynomial tractability implies that the corresponding limit inferior is greater than zero. We will give a complete answer to these questions mentioned above.

The paper is organized as follows. Section 2 contains some basic concepts and summarizes some facts that will be needed in the proof of our main result, which is presented in Section 3.

2. Some concepts and background information

Let $\{H_d\}$ and $\{G_d\}$ be two sequences of normed spaces and for each $d \in \mathbb{N}$ let F_d be a subset of H_d. Assume that we are given a sequence of solution operators

$$S_d : F_d \to G_d \quad \text{for all } d \in \mathbb{N}.$$

For $n \in \mathbb{N}$ and $f \in F_d$, we approximate $S_d f$ by algorithms

$$A_{n,d}(f) = \phi_{n,d}(L_1(f), \dots, L_n(f)),$$

where for each $j \in \{1, 2, \dots, n\}$ the linear functional L_j is from the information class $\Lambda^{\text{all}} = F_d^*$ (where F_d^* denotes the dual space of F_d) and $\phi_{n,d} : \mathbb{R}^n \to G_d$ is an arbitrary mapping. The worst case error $e(A_{n,d})$ of the algorithm $A_{n,d}$ is defined as

$$e(A_{n,d}) = \sup_{f \in F_d} \|S_d(f) - A_{n,d}(f)\|_{G_d}.$$

Furthermore, we define the nth minimal worst-case error as

$$e(n, d) = \inf_{A_{n,d}} e(A_{n,d}),$$

where the infimum is taken over all algorithms using n information operators $L_1, L_2, \dots, L_n$.

For $n = 0$ we use $A_{0,d} = 0$. The error of $A_{0,d}$ is called the initial error and is given by

$$e(0, d) = \sup_{f \in F_d} \|S_d f\|_{G_d}.$$

For $\varepsilon \in (0, 1)$ and $d \in \mathbb{N}$, let $n(\varepsilon, d)$ be the information complexity which is defined as the minimal number of continuous linear functionals which are necessary to obtain an ε-approximation of S_d for the absolute or normalized error criterion, i.e.,

$$n(\varepsilon, d) = \min\{n | e(n, d) \le \varepsilon CRI_d\}, \tag{2.1}$$

where

$$CRI_d = \begin{cases} 1 & \text{for the absolute error criterion,} \\ e(0, d) & \text{for the normalized error criterion.} \end{cases}$$

Next, we list the concepts of tractability below. We say that the problem is

- strongly polynomially tractable (SPT) if there exist non-negative numbers C and p such that

$$n(\varepsilon, d) \le C(\varepsilon^{-1})^p \quad \text{for all } d \in \mathbb{N}, \varepsilon \in (0, 1). \tag{2.2}$$

The exponent p^* of SPT is defined as the infimum of p for which (2.2) holds.