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On construction of blocked general minimum lower-order confounding $2^{n-m}:2^r$ designs with $N/4+1 \le n \le 5N/16$

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ABSTRACT

As an extension of the general minimum lower-order confounding (GMC) criterion to the case of blocked design, a B ^{1}-GMC criterion was proposed (see Wei et al. (2014)). Zhao et al. (2013) constructed all the B ^{1}-GMC $2^{n-m}:2^r$ designs with $5N/16+1 \le n \le N-1$, where $N = 2^{n-m}$, n and r are the numbers of runs, treatment factors and block factors respectively. In this paper, we establish a construction theory to obtain all the B ^{1}-GMC $2^{n-m}:2^r$ designs with $N/4+1 \le n \le 5N/16$. For application, all the B ^{1}-GMC $2^{n-m}:2^r$ designs with $N/4+1 \le n \le 5N/16$ and $N = 16, 32, 64$ and 128 respectively are tabulated.

1. Introduction

Usually, working units in a factorial experiment are not homogeneous. To reduce the systematic errors on the estimation of treatment effects caused by lack of homogeneity, blocking the units into different groups is an efficient method. Thus, how to block the experimental units in a design is an important issue in practice.

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In the last decades, most of the studies on the optimality criterion for blocking were based on the minimum aberration (MA) [8] and clear effects (CE) [14] criteria, such as Sitter, Chen and Feder [11], Chen and Cheng [3], Zhang and Park [20] and Cheng and Wu [5]. Ai and Zhang [1] had a detailed comparison for these blocking criteria. For a summary refer to Mukerjee and Wu [10]. Since Zhang et al. [18] introduced an aliased effect-number pattern (AENP) and GMC criterion, Zhang and Mukerjee [19], Wei et al. [13] and Zhang et al. [17] proposed GMC blocking criteria B-GMC, B^1-GMC and B^2-GMC respectively for different situations.

A main difference between MA and GMC criteria is that, the former is to minimize the average confounding between lower order effects, which is suitable for the situation that all the factors are equally important, while the latter is to minimize the individual confounding between lower order effects, which is suitable when the experimenter has some prior knowledge about ranking factors by importance. So, the optimal designs under B-, B^1- and B^2-GMC criteria are also very useful in practice. Zhao et al. [21] constructed all the B^1-GMC $2^{n-m} : 2^r$ designs with $5N/16 + 1 \le n \le N - 1$ and any r, where $N = 2^{n-m}$ is the number of runs. Tan and Zhang [12] constructed all the B-GMC $2^{n-m} : 2^r$ designs with $5N/16 + 1 \le n \le N - 1$ and $r = 1, 2$. In this paper, we will give a construction theory to obtain all the B^1-GMC $2^{n-m} : 2^r$ designs with $N/4 + 1 \le n \le 5N/16$ and any r.

2. Preliminaries and notation

2.1. Notation and B^1-GMC criterion

Following the notation in [9,21] we denote $q = n - m$ and use the $2^q \times (2^q - 1)$ matrix $H_q = \{\mathbf{1}, \mathbf{2}, \mathbf{21}, \mathbf{3}, \mathbf{31}, \mathbf{32}, \mathbf{321}, \dots, \mathbf{q}\cdots\mathbf{1}\}_{2^q}$ to denote the saturated two-level regular $2^{(2^q-1)-(2^q-1-q)}$ design with Yates order. For simplicity we will omitted the subscript 2^q hereafter. Furthermore, denote $H_1 = F_1 = \{\mathbf{1}\}, H_r = \{H_{r-1}, \mathbf{r}, \mathbf{r}H_{r-1}\}$ and $F_r = H_r \setminus H_{r-1} = \{\mathbf{r}, \mathbf{r}H_{r-1}\}$ for $r = 2, 3, \dots, q$. For a subset $Q \subset H_q$ and a element (column) $\gamma \in H_q$, define $B_2(Q, \gamma) = \#\{(\mathbf{d}_i, \mathbf{d}_j) : \mathbf{d}_i, \mathbf{d}_j \in Q,\ \mathbf{d}_i\mathbf{d}_j = \gamma\}$, where # denotes the cardinality of a set. Say, if $Q = \{\mathbf{2}, \mathbf{3}, \mathbf{31}, \mathbf{32}, \mathbf{321}\}$ is a subset in H_3, then for $\mathbf{1} \in H_3$ we have $B_2(Q, \mathbf{1}) = 2$.

Throughout this paper, we use $D = (D_t : D_b)$ to denote a blocked $2^{n-m} : 2^r$ design taken from H_q, where $\#\{D_t\} = n$, every element in D_t represents a treatment factor, and $D_b \subset H_q \setminus D_t$ with $\#\{D_b\} = 2^r - 1$ is an $(r-1)$-flat of $PG(q-1, 2)$ in finite projective geometry language, which means that in D_b there are r independent block factors and the other columns in D_b are generated by the r independent block factors.

For a regular 2^{n-m} design T, Zhang et al. [18] introduced the sequence

$$^{\#}C(T) = \left({}^{\#}_{1}C_2(T), {}^{\#}_{2}C_2(T), {}^{\#}_{1}C_3(T), {}^{\#}_{2}C_3(T), {}^{\#}_{3}C_2(T), {}^{\#}_{3}C_3(T), \dots\right), \tag{1}$$

called *aliased effect-number pattern* (AENP) of design T, where ${}^{\#}_{i}C_j(T) = ({}^{\#}_{i}C_j^{(0)}(T), {}^{\#}_{i}C_j^{(1)}(T), \dots, {}^{\#}_{i}C_j^{(K_j)}(T))$ and ${}^{\#}_{i}C_j^{(k)}(T)$ is the number of ith-order effects aliased with k jth-order effects and $K_j = \binom{n}{j}$. The design sequentially maximizing the components of the AENP is called a GMC design. Usually there are strings of successive zero components in $^{\#}C(T)$, for shorting notation a string of k 0's is denoted by 0^k and the zeros after the last non-zero component are omitted.

Now all the 2^{n-m} GMC designs with $N/4 + 1 \le n \le N - 1$ were obtained by Li et al. [9], Zhang and Cheng [16] and Cheng and Zhang [6].

To extend the AENP to the case of blocked design, Wei et al. [13] partitioned the $2^n - 1$ treatment effects of a blocked $2^{n-m} : 2^r$ design $D = (D_t : D_b)$ into g-, b-, m- and ϕ-four classes, respectively consisting of those located in the alias cosets containing grand mean, block effect, treatment main effect and none of the three kinds of effects. They then introduced the set

$$\left\{{}^{\#}{}^{g}_{i}C_j(D),\ {}^{\#}{}^{b}_{i}C_j(D),\ {}^{\#}{}^{m}_{i}C_j(D),\ {}^{\#}{}^{\phi}_{i}C_j(D),\ i, j = 1, \dots, n\right\} \tag{2}$$

and called it *blocked aliased effect number pattern* (B-AENP) of design D, where ${}^{\#}{}^{*}_{i}C_j(D) = ({}^{\#}{}^{*}_{i}C_j^{(0)}(D), {}^{\#}{}^{*}_{i}C_j^{(1)}(D), \dots, {}^{\#}{}^{*}_{i}C_j^{(K_j)}(D))$ and ${}^{\#}{}^{*}_{i}C_j^{(k)}(D)$ is the number of ith-order effects aliased with k jth-order