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Geometric isomorphism check for symmetric factorial designs

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ABSTRACT

Two designs are geometrically isomorphic if one design can be obtained from the other by reordering the runs, relabeling the factors and/or reversing the level order of one or more factors. In this paper, some new necessary and sufficient conditions for identifying geometric isomorphism of symmetric designs with prime levels are provided. A new algorithm for checking geometric isomorphism is proposed and a searching result for geometrically non-isomorphic 3-level orthogonal arrays of 18 runs is presented.

1. Introduction

Factorial designs are commonly used in various fields. In such an application, a number of fixed levels are selected for each factor and then some level-combinations are chosen to be the runs in an experiment. A factor can be either qualitative or quantitative. An important problem in practice is the choice of optimal factorial designs. Optimal designs are naturally expected to be identified according to some design criterion from a set of candidate designs. To assure that the optimal design is indeed the global optimal one, the candidate set is usually very large, and even infinite for designs with quantitative factors if we do not impose some discretization. The computer search is exhaustive. To save time, one needs to tell whether two designs are in fact "equal" or not. For qualitative factors, two designs are said to be equivalent or *combinatorially isomorphic*, if one design can be obtained from the other by reordering the runs, relabeling the factors and/or switching the levels of one or more factors. Since combinatorially isomorphic designs share the same statistical properties in the classical ANOVA model and are essentially the same, we need only to consider one of them in any search for optimal designs to avoid burdensome computations. However, the ANOVA model is not suitable for a design with quantitative factors, which aims to fit a model that indicates the relationship between the factors and response. Cheng and Wu [5] reported that level permutations of a 3^{n-k} design

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could lead to different model efficiencies when a polynomial model is fitted and defined the "model isomorphism". Cheng and Ye [4] pointed out that such a model non-isomorphism is the result of different geometric structures caused by permuting the levels of factors. Furthermore, they defined geometric isomorphism for two designs with quantitative factors. Two designs are *geometrically isomorphic* if one design can be obtained from the other by reordering the runs, relabeling the factors and/or reversing the level order of one or more factors.

For two-level designs, combinatorial isomorphism is equivalent to geometric isomorphism and has been studied extensively in the literature. Draper and Mitchell [7,8] proved that having the same word-length pattern is necessary for the equivalence of two such designs. Draper and Mitchell [9] and Chen and Lin [2] showed that the letter pattern, which counts the frequencies of letters contained in the defining words of different lengths, does not uniquely determine a two-level design. Chen et al. [3] proposed a comprehensive algorithm for detecting combinatorially isomorphic designs beyond the comparisons of word-length patterns and letter patterns. Clark and Dean [6] presented a method of determining isomorphism of any two factorial designs by examining the Hamming distance matrices of their projection designs and provided an algorithm for checking the isomorphism of two-level fractional factorial designs. Ma et al. [13] proposed an algorithm for detecting the combinatorial isomorphism of two-level and high-level designs. Lin and Sitter [11] created a new isomorphism check and constructed a complete catalog of non-isomorphic 2^{k-p} designs, the final step in their isomorphism check algorithm refers to that of Clark and Dean [6]. Recently, Liu et al. [12] proposed a three-dimensional matrix named LIPM and showed that LIPM is an efficient tool for the isomorphism check of regular two-level fractional factorial designs. However, necessary and sufficient conditions for identifying geometric isomorphism had not been provided until Clark and Dean [6] gave some discussions. Cheng and Ye [4] developed another one based on indicator functions in polynomial forms.

This paper aims at providing some new necessary and sufficient conditions and an algorithm for identifying geometric isomorphism and is organized as follows. Section 2 gives some preliminary notation and results before the main results are presented. In Section 3, we propose some new necessary and sufficient conditions for identifying geometric isomorphism of symmetric designs with prime levels. In Section 4, a new algorithm for checking geometric isomorphism is proposed and a searching result for geometrically non-isomorphic 3-level orthogonal arrays of 18 runs is presented. Section 5 points out that some of the theoretical results are suitable for any symmetric design, whatever levels it has.

2. Preliminary notation and results

A symmetric factorial design of N runs and n factors with p levels is an $N \times n$ matrix with entries from a set of p symbols and denoted by (N, p^n). Except in Section 5, we assume that p is a prime greater than 2 and the p levels are taken to be $(-p+1)/2, (-p+3)/2, \ldots, (p-1)/2$. If the p^t possible level-combinations appear equally often for any t columns of an (N, p^n)-design, the design is said to be an orthogonal array with strength t and denoted by $OA(N, p^n, t)$. When $t = 1$, the design is called balanced, and when $t \geq 2$, the columns of the design are said to be orthogonal with each other.

Consider the following equivalence classes partitioning the set $\mathcal{Z}$ of integers:

$$[(-p+1)/2] = \{(-p+1)/2 + pk, k \in \mathcal{Z}\},$$
$$[(-p+3)/2] = \{(-p+3)/2 + pk, k \in \mathcal{Z}\},$$
$$\ldots$$
$$[(p-1)/2] = \{(p-1)/2 + pk, k \in \mathcal{Z}\}.$$

We define two binary operations on the set $\mathcal{E}_p = \{[(-p+1)/2+i],\ i = 0, \ldots, p-1\}$ of the equivalence classes as

$$[a] + [b] = [a+b], \qquad [a] \cdot [b] = [a \cdot b],$$

where a is an element of set $[a]$, so is b, and the sum $a + b$ and product $a \cdot b$ are the ordinary sum and product of a and b, respectively. Obviously, the set $\mathcal{E}_p$ forms a finite field, in which the identity elements for the operation "+" and "·" are [0] and [1], respectively.