Journal of Complexity 28 (2012) 459–467

Contents lists available at SciVerse ScienceDirect

Journal of Complexity

journal homepage: www.elsevier.com/locate/jco

On optimal two-level nonregular factorial split-plot designs

Xue-Min Zi [a,b], Runchu Zhang [b,c], Min-Qian Liu [b,*]

[a] School of Science, Tianjin University of Technology and Education, Tianjin 300222, China

[b] Department of Statistics, School of Mathematical Sciences and LPMC, Nankai University, Tianjin 300071, China

[c] KLAS and School of Mathematics and Statistics, Northeast Normal University, Changchun 130024, China

ARTICLE INFO

Article history:
Received 28 August 2011
Accepted 2 February 2012
Available online 25 February 2012

Keywords:
Hadamard matrix
Indicator function
Nonregular
Split-plot design

ABSTRACT

This article studies two-level nonregular factorial split-plot designs. The concepts of indicator function and aliasing are introduced to study such designs. The minimum *G*-aberration criterion proposed by Deng and Tang (1999) [4] for two-level nonregular factorial designs is extended to the split-plot case. A method to construct the whole-plot and sub-plot parts is proposed for nonregular designs. Furthermore, the optimal split-plot schemes for 12-, 16-, 20- and 24-run two-level nonregular factorial designs are searched, and many such schemes are tabulated for practical use.

1. Introduction

A split-plot design is often used when it is not practical to perform all the experimental runs of a multifactorial experiment in a completely random order. Recently, many authors have focused on fractional factorial split-plot (FFSP) designs, see e.g., [8,1,19,15,17,16,3] and the references therein. To perform an FFSP design with m factors, we often first randomly choose one of the factorial level-settings of these, say m_1, hard-to-change factors and then run all of the level-combinations of the remaining $m_2(= m - m_1)$ factors in a random order, while holding the m_1 factors fixed. This is repeated for each level-combination of these m_1 factors. If the design matrix for this experimental setup is identical to a 2^{m-k} fractional factorial (FF) design, where $m = m_1 + m_2$ and $k = k_1 + k_2$, then it is said to be a $2^{(m_1+m_2)-(k_1+k_2)}$ FFSP design. The m_1 and m_2 factors are called *whole-plot* (WP) and *sub-plot* (SP) factors, respectively. There are k_1 WP and k_2 SP fractional generators. The group formed by the $k = k_1 + k_2$ generators is called the defining contrast subgroup. Let A_i denote the number of words of length i in the defining contrast subgroup of a $2^{(m_1+m_2)-(k_1+k_2)}$ design, then the vector $W = (A_3, \ldots, A_m)$ is called the word-length pattern of the design. The maximum resolution and minimum aberration (MA) criteria can then be defined [8,1].

* Corresponding author.
E-mail address: mqliu@nankai.edu.cn (M.-Q. Liu).

0885-064X/$ – see front matter
doi:10.1016/j.jco.2012.02.003

All the papers mentioned above discussed only split-plot schemes for regular FF designs. However, nonregular factorial designs have some advantages over regular ones in terms of run size flexibility and estimation capacity. Therefore, they are becoming popular choices in practice, and in many situations they need to possess a split-plot structure. In this article, we extend the minimum G-aberration (MGA) criterion proposed by Deng and Tang [4] for two-level nonregular factorial designs to the split-plot case, and provide a method for constructing two-level nonregular factorial split-plot designs from Hadamard matrices.

2. Indicator function and aliasing

In this section, a polynomial representation for general two-level factorial designs is presented. It applies to any two-level factorial design, with or without replicates, regular or nonregular, and can set up the mathematical framework for studying nonregular factorial split-plot designs.

2.1. Indicator function

Let F be a 2^m full factorial design. Without loss of generality, the levels of each factor in F are denoted by 1 and -1. We use an $n \times m$ matrix of 1 and -1 to represent a factorial design D, where each row of the matrix corresponds to a run and each column to a factor. According to Refs. [18,2], we have

Definition 1. A factorial design D corresponds to a unique polynomial function defined on F with the form

$$F_D(x_1, \dots, x_m) = h_0 + \sum_{k=1}^{m} \sum_{1 \le i_1 < \cdots < i_k \le m} h_{i_1 \cdots i_k} x_{i_1} \cdots x_{i_k}, \tag{1}$$

where $h_0 = n/2^m$, $h_{i_1 \cdots i_k} = \frac{1}{2^m} \sum_{X \in D} x_{i_1} \cdots x_{i_k}$, and $X = (x_1, \dots, x_m)$ represents a design point in F. The summation $\sum_{X \in D} x_{i_1} \cdots x_{i_k}$ can be viewed as a general inner product of k columns of D. The polynomial function (1) is called the *indicator function* of D, the polynomial terms appearing in (1) (i.e., those polynomial terms with nonzero coefficients) are called the *words* of D, and these words form the *defining contrast subgroup* of D.

The indicator function approach can be generalized to factorial split-plot designs directly. A distinction between a completely randomized design and a factorial split-plot design is that in the latter there are two types of factors, WP factors and SP factors. Now let us see two illustrative examples. The first one is modified from an example given by Montgomery [12, p. 307] for the purpose of the illustration.

Example 1. Suppose we wish to perform an experiment to identify factor settings that will improve the efficiency of a ball mill. Engineers have identified six potentially important factors, each at two levels: motor speed X_1, feed mode X_2, feed sizing X_3, material type X_4, gain X_5, and screen angle X_6. Suppose that it is expensive or time consuming to change the levels of X_1, X_2, X_3 and X_4, and there are only enough resources to perform 16 experimental runs. Let the defining contrast subgroup for a regular FFSP design D_1 be $I = X_1X_2X_3X_4 = X_2X_3X_5X_6 = X_1X_4X_5X_6$, that is, D_1 is a $2^{(4+2)-(1+1)}$ FFSP design with $X_4 = X_1X_2X_3$ as the WP part and $X_6 = X_2X_3X_5$ as the SP part. The WP part for this experiment is a 2^{4-1} FF design, while the SP part is a design with generator $X_2X_3X_5X_6$, selected from the interactions of WP and SP factors. For D_1, the word-length pattern is (0, 3, 0, 0), and its indicator function is:

$$\begin{aligned} F_{D_1}(x_1, \dots x_6) &= \frac{1}{4}(1 + x_1x_2x_3x_4)(1 + x_2x_3x_5x_6) \\ &= \frac{1}{4}(1 + x_1x_2x_3x_4 + x_2x_3x_5x_6 + x_1x_4x_5x_6). \end{aligned} \tag{2}$$

For any $X = (x_1, \dots, x_6) \in D_1$, $F_{D_1}(X) = 1$ because $x_1x_2x_3x_4 = 1$ and $x_1x_4x_5x_6 = 1$, while for any $X \in F \setminus D_1$ with $F = 2^6$, we have $F_{D_1}(X) = 0$, since either $x_1x_2x_3x_4 = -1$ or $x_1x_4x_5x_6 = -1$. Therefore, the polynomial in (2) determines the design D_1.