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Dissipative boundary conditions for 2×2 hyperbolic systems of conservation laws for entropy solutions in BV [☆]

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Abstract

In this article, we investigate the BV stability of 2×2 hyperbolic systems of conservation laws with strictly positive velocities under dissipative boundary conditions. More precisely, we derive sufficient conditions guaranteeing the exponential stability of the system under consideration for entropy solutions in BV. Our proof is based on a front tracking algorithm used to construct approximate piecewise constants solutions whose BV norms are controlled through a Lyapunov functional. This Lyapunov functional is inspired by the one proposed in J. Glimm's seminal work [16], modified with some suitable weights in the spirit of the previous works [10,9].

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Contents

1. Introduction

1.1. Setting and main result

The goal of this article is to study the exponential stability of 2×2 systems of conservation laws on a finite interval, by means of boundary feedbacks, in the context of weak entropy solutions. To be more precise, we consider the following setting: Let Ω be an open subset of $\mathbb{R}^2$ with $0 \in \Omega$, and $f : \Omega \mapsto \mathbb{R}^2$ be a smooth function (supposed to satisfy the strict hyperbolicity condition described below) and consider the following system of two conservation laws

$$\partial_t u + \partial_x (f(u)) = 0 \quad \text{for } (t,x) \in (0,\infty) \times (0,L). \tag{1.1}$$

In System (1.1), the solution $u = u(t,x) = (u_1, u_2)^T$ has 2 components and the space variable x belongs to the finite interval $(0, L)$. We assume the flux function f to satisfy the strictly hyperbolicity conditions, that is,

$$\forall u \in \Omega, \quad \text{the matrix } A(u) = Df(u) \text{ has 2 real distinct eigenvalues } \lambda_1(u) < \lambda_2(u). \tag{1.2}$$

Furthermore, we make the assumption that both velocities are positive, so that we finally get:

$$0 < \lambda_1(u) < \lambda_2(u) \quad \text{for all } u \in \Omega. \tag{1.3}$$