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Well-posedness on a new hydrodynamic model of the fluid with the dilute charged particles

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Abstract

We use an energetic variational approach to derive a new hydrodynamic model, which could be called a generalized Poisson–Nernst–Planck–Navier–Stokes system. Such the system could describe the dynamics of the compressible conductive fluid with the dilute charged particles and be used to analyze the interactions between the macroscopic fluid motion and the microscopic charge transportation. Then, we develop a general method to obtain the unique local classical solution, the unique global solution under small perturbations and the optimal decay rates of the solution and its derivatives of any order.
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1. Introduction

In real life, there are many fluids with the charged particles which transport under the influence of the (self-)electrostatic potential. For instance, the flow in semi-conductors [29,39,47,51], ion channels [8,23,48], plasmas [10,12,53], and so on. Then, some mathematical models were proposed to describe and simulate the dynamics of such the fluids in light of different considerations, such as, the Vlasov–Poisson–Boltzmann model [7,36,39], the hydrodynamic model (for example, Euler–Poisson model, Navier–Stokes–Poisson model, etc.) [5,9,29,31], the Poisson–Nernst–Planck (PNP) model (also called drift-diffusion or electro-diffusion model) [8,29,33], etc.

In this paper, we use an Energetic Variational Approach (EVA) to explore a new hydrodynamical model in $\mathbb{R}^3$:

$$\begin{cases} \rho_t + \operatorname{div}(\rho u) = 0, \\ (\rho u)_t + \operatorname{div}(\rho u \otimes u) + \nabla p(\rho) \\ \quad = \mu \Delta u + (\mu + \mu')\nabla \operatorname{div} u - \nabla\varphi(v) - \nabla\psi(w) + \epsilon \Delta\phi \nabla\phi, \\ v_t + \operatorname{div}(vu) = \operatorname{div}\left(D_v \nabla\varphi(v) + D_v z_v e v \nabla\phi\right), \\ w_t + \operatorname{div}(wu) = \operatorname{div}\left(D_w \nabla\psi(w) + D_w z_w e w \nabla\phi\right), \\ -\epsilon\Delta\phi = z_v e v + z_w e w, \end{cases} \tag{1.1}$$

which could be called a generalized Poisson–Nernst–Planck–Navier–Stokes (PNP-NS) system since it seems that the (microscopic) PNP system couples with the (macroscopic) compressible NS system. Here, the unknown variables ρ, u, $v(w)$, ϕ and p represent the fluid density, the fluid velocity, the distribution of the negative (positive) charge, the electrostatic potential and the pressure function, respectively. The functions φ, ψ are determined by the entropy density σ, h given in (2.4) in light of the relations (2.8) and (2.11). The physical parameters are the viscosity coefficients of fluid μ, μ', the dielectric constant ϵ, the charge of one electron e, the diffusion coefficient of negative (positive) ions $D_v(D_w)$ and the valence of negative (positive) ions $z_v(z_w)$.

Now, we review the history about the Poisson–Nernst–Planck (PNP) system. The original PNP model can be read as the following diffusion equations

$$\begin{cases} v_t = \operatorname{div}\left[D_v\left(\nabla v + \dfrac{z_v e}{k_B T} v \nabla\phi\right)\right], \\ w_t = \operatorname{div}\left[D_w\left(\nabla w + \dfrac{z_w e}{k_B T} w \nabla\phi\right)\right], \\ -\epsilon\Delta\phi = z_v e v + z_w e w, \end{cases} \tag{1.2}$$

with the corresponding energy dissipation law

$$\begin{aligned} \frac{d}{dt}E^{total} &:= \frac{d}{dt}\int k_B T\left(v\ln v + w\ln w\right) + \frac{1}{2}\phi(z_v e v + z_w e w) \\ &= -\int \frac{k_B T}{D_v} v\,|u_v|^2 + \frac{k_B T}{D_w} w\,|u_w|^2 := -\triangle. \end{aligned} \tag{1.3}$$

The above k_B, T and $u_v(u_w)$ denote the Boltzmann constant, the absolute temperature and the effective velocity of the negative (positive) charge, respectively. Then Hsieh et al. [27] derived

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