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Existence of traveling waves with the critical speed for a discrete diffusive epidemic model ☆

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Abstract

We study the traveling wave solutions for a discrete diffusive epidemic model of classical Kermack–McKendrik type. The existence of traveling waves with super-critical speeds are well-known. By a delicate analysis of traveling waves with super-critical speeds, we derive the existence of traveling waves with the critical speed.

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1. Introduction

In this paper, we consider the following lattice dynamical system

$$\begin{cases} \dot{S}_j := \dfrac{dS_j}{dt} = d(S_{j+1} - 2S_j + S_{j-1}) - \beta S_j I_j, & j \in \mathbb{Z}, \\ \dot{I}_j := \dfrac{dI_j}{dt} = (I_{j+1} - 2I_j + I_{j-1}) + (\beta S_j - \gamma) I_j, & j \in \mathbb{Z}, \end{cases} \tag{1.1}$$

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where d, β, γ are positive constants. System (1.1) is the well-known classical Kermack–McKendrik epidemic model ([7]) describing an infectious disease outbreak in a closed population in which the environment is of one-dimensional and is divided into countably discrete niches. Here $S_j(t)$ is the susceptible population and $I_j(t)$ is the infected population at niches j at time t, respectively, β is the transmission coefficient and γ is the recovery/remove rate per unit time. We also assume the migration coefficients of susceptible and infective populations are d and 1, respectively.

To see whether a disease can propagate spatially with a constant speed, one usually look for the so-called traveling wave solutions defined as follows.

For a given $s^* > 0$, by a traveling wave solution of system (1.1), we mean a solution of system (1.1) in the form

$$(S_j(t), I_j(t)) = (S(\xi), I(\xi)), \quad \xi = j + ct,$$

such that $0 < S < s^*$ and $I > 0$ in $\mathbb{R}$, $S(-\infty) = s^*$ and $I(\pm\infty) = 0$. Here the constant c (the wave speed) and the function (S, I) (the wave profile) are to be determined. It is easy to see that (c, S, I) satisfies the system

$$cS' = dD[S] - \beta SI \quad \text{in } \mathbb{R}, \tag{1.2}$$

$$cI' = D[I] + (\beta S - \gamma)I \quad \text{in } \mathbb{R}, \tag{1.3}$$

where $D[\phi](\xi) := \phi(\xi + 1) + \phi(\xi - 1) - 2\phi(\xi)$. Note that we only require the boundary conditions

$$(S, I)(-\infty) = (s^*, 0), \quad I(+\infty) = 0,$$

and leave the value of S at $+\infty$ to be free.

There are tremendous works devoted to the study of traveling waves for different variants of Kermack–McKendrik model and other epidemic models (see the nice survey paper [6]). We also refer the reader to, for example, [4,11,10,12,9,8] for the studies of continuous reaction–diffusion models and [5,3] for the discrete models. One should notice that the system (1.2)–(1.3) is nonlocal and non-monotone.

For a given $s^* > \gamma/\beta$, we let

$$c^* := \inf_{\lambda > 0} \frac{(e^{\lambda} + e^{-\lambda} - 2) + (\beta s^* - \gamma)}{\lambda}. \tag{1.4}$$

It was proved in [5] that, under certain restrictions on s^*, there exists a traveling wave solution of (1.1) for any speed $c > c^*$. Also, the non-existence of traveling waves for $c < c^*$ when $s^* > \gamma/\beta$ and any $c > 0$ when $0 < s^* \le \gamma/\beta$ was also derived. The question for the existence of traveling waves for $c = c^*$ was left open in [5].

The main purpose of this work is to derive the existence of traveling wave solutions for the critical speed c^*. Moreover, we shall remove the restrictions on s^* for the existence of traveling waves for $c > c^*$. Our method is motivated by the recent work [3] in which the following lattice dynamical system for the classical endemic model was studied