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Uniform boundary regularity in almost-periodic homogenization

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Abstract

In the present paper, we generalize the theory of quantitative homogenization for second-order elliptic systems with rapidly oscillating coefficients in $APW^2(\mathbb{R}^d)$, which is the space of almost-periodic functions in the sense of H. Weyl. We obtain the large scale uniform boundary Lipschitz estimate, for both Dirichlet and Neumann problems in $C^{1,\alpha}$ domains. We also obtain large scale uniform boundary Hölder estimates in $C^{1,\alpha}$ domains and L^2 Rellich estimates in Lipschitz domains.

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Keywords: Homogenization; Elliptic system; Boundary regularity

Contents

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1. Introduction

This paper is a continuation of our previous work [23] and generalizes the global uniform Lipschitz estimate in periodic or uniformly almost-periodic homogenization to the second-order elliptic operators with coefficients in a broader class of discontinuous almost-periodic functions. Precisely we will study a family of elliptic operators with rapidly oscillating almost-periodic coefficients in the form of

$$\mathcal{L}_\varepsilon = -\mathrm{div}(A(\cdot/\varepsilon)\nabla) = -\frac{\partial}{\partial x_i}\left\{a_{ij}^{\alpha\beta}\left(\frac{\cdot}{\varepsilon}\right)\frac{\partial}{\partial x_j}\right\}, \qquad \varepsilon > 0 \tag{1.1}$$

where summation convention is used throughout and ε is assumed to be a tiny parameter. We will assume that the coefficient matrix $A(y) = (a_{ij}^{\alpha\beta}(y))$ with $1 \le i, j \le d$ and $1 \le \alpha, \beta \le m$ is real, bounded, measurable, and satisfies the following conditions:

(i) Strong ellipticity: for some $\mu > 0$, and all $y \in \mathbb{R}^d$ and $\xi = (\xi_i^\alpha) \in \mathbb{R}^{d\times m}$,

$$\mu|\xi|^2 \le a_{ij}^{\alpha\beta}(y)\xi_i^\alpha\xi_j^\beta \le \mu^{-1}|\xi|^2. \tag{1.2}$$

(ii) Almost-periodicity in the sense of H. Weyl (1927): each entry of A may be approximated by a sequence of trigonometric polynomials with respect to the semi-norm

$$\|f\|_{W^2} = \limsup_{R\to\infty}\sup_{x\in\mathbb{R}^d}\left(\fint_{B(x,R)}|f|^2\right)^{1/2}. \tag{1.3}$$

In this situation, we also say $A \in APW^2(\mathbb{R}^d)$. We emphasize that this class of almost-periodic functions, which allows discontinuous functions, is much broader than that of uniformly almost-periodic functions in the sense of H. Bohr (1925) considered in [21,4,1], which is the closure of trigonometric polynomials with respect to the L^∞ norm [8].

We consider the following Dirichlet problem (DP) in a bounded domain Ω:

$$\mathcal{L}_\varepsilon(u_\varepsilon) + \lambda u_\varepsilon = F \quad \text{in } \Omega, \qquad \text{and} \qquad u_\varepsilon = f \quad \text{on } \partial\Omega, \tag{1.4}$$