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Transition fronts in time heterogeneous and random media of ignition type

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Abstract

The current paper is devoted to the investigation of wave propagation phenomenon in reaction–diffusion equations with ignition type nonlinearity in time heterogeneous and random media. It is proven that such equations in time heterogeneous media admit transition fronts with time dependent profiles and that such equations in time random media admit transition fronts with random profiles. Important properties of transition fronts, including the boundedness of propagation speeds and the uniform decaying estimates of the propagation fronts, are also obtained.

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Keywords: Transition front; Time heterogeneous media; Time random media

Contents

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1. Introduction

Consider the one-dimensional reaction–diffusion equation

$$u_t = u_{xx} + f(t, x, u), \quad x \in \mathbb{R},\ t \in \mathbb{R}, \tag{1.1}$$

where $f(t, x, u)$ is of ignition type, that is, there exists $\theta \in (0, 1)$ such that for all $t \in \mathbb{R}$ and $x \in \mathbb{R}$, $f(t, x, u) = 0$ for $u \in [0, \theta] \cup \{1\}$ and $f(t, x, u) > 0$ for $u \in (\theta, 1)$. Such an equation arises in the combustion theory (see e.g. [8,10]). The number θ is usually referred to as the ignition temperature. The front propagation concerning this equation was first investigated by Kanel (see [14–17]) in the space–time homogeneous media, i.e., $f(t, x, u) = f(u)$; he proved that all solutions, with initial data in some subclass of continuous functions with compact support and values in $[0, 1]$, propagate at the same speed $c_* > 0$, which is the speed of the unique traveling wave solution $\psi(x - c_*t)$, where ψ satisfies

$$\psi_{xx} + c_*\psi_x + f(\psi) = 0, \quad \lim_{x\to-\infty} \psi(x) = 1 \quad \text{and} \quad \lim_{x\to\infty} \psi(x) = 0.$$

Also see [3,4,11,12] and references therein for the treatment of traveling wave solutions of (1.1) in space–time homogeneous media.

Recently, equation (1.1) in the space heterogeneous media, i.e., $f(t, x, u) = f(x, u)$, has attracted a lot of attention. In terms of space periodic media, that is, $f(x, u)$ is periodic in x, Berestycki and Hamel proved in [5] the existence of pulsating fronts or periodic traveling waves of the form $\psi(x - c_*t, x)$, where $\psi(s, x)$ is periodic in x and satisfies a degenerate elliptic equation with boundary conditions $\lim_{s\to-\infty} \psi(s, x) = 1$ and $\lim_{s\to\infty} \psi(s, x) = 0$ uniformly in x. In the work of Weinberger (see [33]), he proved from the dynamical system viewpoint that solutions with general non-negative compactly supported initial data spread with the speed c_*. We also refer to [34–36] for related works.

In the general space heterogeneous media, wavefront with a profile is no longer appropriate, and we are looking for more general wavefronts such as transition fronts in the sense of Berestycki and Hamel (see [6,7]), that is,

Definition 1.1. A global-in-time solution $u(t, x)$, $x \in \mathbb{R}$, $t \in \mathbb{R}$ of (1.1) is called a transition front if there is a function $\xi : \mathbb{R} \to \mathbb{R}$, called, interface location function, such that

$$u(t, x) \to 1 \text{ uniformly in } t \text{ and } x \le \xi(t) \text{ as } x - \xi(t) \to -\infty, \text{ and}$$
$$u(t, x) \to 0 \text{ uniformly in } t \text{ and } x \ge \xi(t) \text{ as } x - \xi(t) \to \infty.$$