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ScienceDirect
J. Differential Equations 262 (2017) 590–614
Journal of Differential Equations
www.elsevier.com/locate/jde

Determining the first order perturbation of a polyharmonic operator on admissible manifolds

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Received 17 August 2015; revised 26 April 2016
Available online 30 September 2016

Abstract

We consider the inverse boundary value problem for the first order perturbation of the polyharmonic operator $\mathcal{L}_{g,X,q}$, with X being a $W^{1,\infty}$ vector field and q being an L^∞ function on compact Riemannian manifolds with boundary which are conformally embedded in a product of the Euclidean line and a simple manifold. We show that the knowledge of the Dirichlet-to-Neumann map determines X and q uniquely. The method is based on the construction of complex geometrical optics solutions using the Carleman estimate for the Laplace–Beltrami operator due to Dos Santos Ferreira, Kenig, Salo and Uhlmann. Notice that the corresponding uniqueness result does not hold for the first order perturbation of the Laplace–Beltrami operator.

1. Introduction

Let (M, g) be a compact oriented Riemannian smooth manifold with boundary. Throughout this paper, the word "smooth" will be used as the synonym of "C^∞". Let Δ_g be the Laplace–Beltrami operator associated to the metric g which is given in local coordinates by

$$\Delta_g u = |g|^{-1/2} \frac{\partial}{\partial x^j} \left(|g|^{1/2} g^{jk} \frac{\partial u}{\partial x^k} \right),$$

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http://dx.doi.org/10.1016/j.jde.2016.09.039
0022-0396/

where as usual (g^{jk}) is the matrix inverse of (g_{jk}), and $|g| = \det(g_{jk})$. If F denotes a function or distribution space (C^k, L^p, H^k, $\mathcal{D}'$, etc.), then we will denote by $F(M, TM)$ the corresponding space of vector fields on M.

Let $X \in W^{1,\infty}(M, TM)$ and $q \in L^\infty(M)$. Consider the polyharmonic operator $(-\Delta_g)^m$, $m \geq 1$, with the first order perturbation induced by X and q

$$\mathcal{L}_{g,X,q} = (-\Delta_g)^m + X + q.$$

The operator $\mathcal{L}_{g,X,q}$ equipped with the domain

$$\mathcal{D}(\mathcal{L}_{g,X,q}) = \{u \in H^{2m}(M) : \gamma u = 0\} = H^{2m}(M) \cap H_0^m(M)$$

is an unbounded closed operator on $L^2(M)$ with purely discrete spectrum; see [8]. Here and in what follows,

$$\gamma u := (u|_{\partial M}, \Delta_g u|_{\partial M}, \dots, \Delta_g^{m-1} u|_{\partial M})$$

is the Dirichlet trace of u, and $H^s(M)$ is the standard Sobolev space on M, $s \in \mathbb{R}$.

We make the assumption that 0 is not a Dirichlet eigenvalue of $\mathcal{L}_{g,X,q}$ in M. Under this assumption, for any $f = (f_0, \dots, f_{m-1}) \in \mathcal{H}_m(\partial M) := \prod_{j=0}^{m-1} H^{2m-2j-1/2}(\partial M)$, the Dirichlet problem

$$\begin{aligned} \mathcal{L}_{g,X,q} u &= 0 \quad \text{in} \quad M, \\ \gamma u &= f \quad \text{in} \quad \partial M, \end{aligned} \tag{1}$$

has a unique solution $u \in H^{2m}(M)$. Let ν be an outer unit normal to ∂M. Introducing the Neumann trace operator $\widetilde{\gamma}$ by

$$\widetilde{\gamma} : H^{2m}(M) \to \prod_{j=0}^{m-1} H^{2m-2j-3/2}(\partial M),$$

$$\widetilde{\gamma} u = (\partial_\nu u|_{\partial M}, \partial_\nu \Delta_g u|_{\partial M}, \dots, \partial_\nu \Delta_g^{m-1} u|_{\partial M}),$$

we define the Dirichlet-to-Neumann map $N_{g,X,q}$ by

$$N_{g,X,q} : \mathcal{H}_m(\partial M) \to \prod_{j=0}^{m-1} H^{2m-2j-3/2}(\partial M), \quad N_{g,X,q}(f) = \widetilde{\gamma} u,$$

where $u \in H^{2m}(M)$ is the unique solution to the boundary value problem (1). Let us also introduce the set of the Cauchy data for the operator $\mathcal{L}_{g,X,q}$

$$C_{g,X,q} = \{(\gamma u, \widetilde{\gamma} u) : u \in H^{2m}(M), \quad \mathcal{L}_{g,X,q} u = 0\}.$$

When 0 is not a Dirichlet eigenvalue of $\mathcal{L}_{g,X,q}$ in M, the set $C_{g,X,q}$ is the graph of the Dirichlet-to-Neumann map $N_{g,X,q}$.