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On the existence of local strong solutions to chemotaxis–shallow water system with large data and vacuum

Jiahang Che [a], Li Chen [a], Ben Duan [a,b], Zhen Luo [c,*]

[a] School of Business Informatics and Mathematics, University of Mannheim, Mannheim, 68131, Germany
[b] School of Mathematics, Dalian University of Technology, Dalian, 116024, People's Republic of China
[c] School of Mathematical Sciences, Xiamen University, Xiamen, 361005, People's Republic of China

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Abstract

In this paper, motivated by the chemotaxis–Navier–Stokes system arising from mathematical biology [43], a modified shallow water type chemotactic model is derived. For large initial data allowing vacuum, the local existence of strong solutions together with the blow-up criterion is established.

MSC: primary 35Q35, 35Q92, 76N10, 92C17; secondary 35M10, 35Q30

Keywords: Strong solution; Chemotaxis–fluid interaction; Shallow water system; Vacuum; A priori estimate

1. Introduction and main results

A mathematical model was proposed in [43] for bacteria cells living in a viscous fluid, where the process is under the influence of convective fluid transportation, the gravitational force and the

* Corresponding author.
E-mail addresses: che@math.uni-mannheim.de (J. Che), chen@math.uni-mannheim.de (L. Chen), bduan@dlut.edu.cn, bduan@mail.uni-mannheim.de (B. Duan), zluo@xmu.edu.cn (Z. Luo).

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chemotactic movement driven by biological signals. In this paper, we consider the chemotaxis–shallow water system

$$\begin{cases} n_t + \operatorname{div}(n\boldsymbol{u}) = D_n \Delta n - \nabla \cdot (n\chi(c)\nabla c), \\ c_t + \operatorname{div}(c\boldsymbol{u}) = D_c \Delta c - nf(c), \\ h_t + \operatorname{div}(h\boldsymbol{u}) = 0, \\ h\boldsymbol{u}_t + h\boldsymbol{u} \cdot \nabla \boldsymbol{u} + h^2 \nabla n + \dfrac{1}{2}(1+n)\nabla h^2 = \mu \Delta \boldsymbol{u} + (\mu + \lambda)\nabla(\operatorname{div}\boldsymbol{u}), \end{cases} \tag{1.1}$$

which is derived from the chemotaxis–Navier–Stokes equations in [43]. Here, the unknowns are n, c, h, $\boldsymbol{u}$ presenting bacterial density, substrate concentration, the fluid height and the fluid velocity field, respectively. $\Omega \subset \mathbb{R}^2$ is the physical domain where the cells and fluid move and interact. Constants D_n and D_c are the corresponding diffusion coefficients for the cells and substrate. The chemotactic sensitivity $\chi(c)$ and the consumption rate of the substrate by the cells $f(c)$ are supposed to be given smooth functions. The constants μ and λ are the shear viscosity and the bulk viscosity coefficients respectively with the following physical restrictions:

$$\mu > 0, \quad \mu + \lambda \geq 0.$$

Before getting into details on the derivation and mathematical analysis of (1.1), related mathematical results on chemotactic models in biomathematics and shallow water system in fluid dynamics will be outlined.

Chemotaxis is a well-known biological phenomenon describing the collective motion of cells or the evolution of density of bacteria driven by chemicals, such as cell migration, formation of organs, cancer progress (and etc.). In the last few decades, scientists developed mathematical models for chemotaxis, among which the best-studied one is the Patlak–Keller–Segel system [27,28,38]. After the first existence and blow-up results in [25], mathematical analysis on chemotactic models attracted many mathematicians to work in this field. The reason why this type of system is interesting is that it induced two different mechanisms, namely diffusion and aggregation. A large series of results have been obtained for a phenomenon called chemotactic collapse, which was originally conjectured in [11,37], i.e., there exists a threshold of critical mass for global existence and finite-time blow-up. One may refer to [5,21,39,41,44] for more details. For multi-dimensional Patlak–Keller–Segel system with degenerate diffusion, the threshold was established in [1,7,8,24,45]. For the parabolic–parabolic Keller–Segel model and kinetic models for chemotaxis, interested readers can refer to [2,6,10] and the references therein.

The evolution of an incompressible fluid in three space dimensions in response to gravitational and rotational accelerations can be simulated by the non-linear shallow water equations. The solutions were studied in [20,40,42] with the initial data close to a constant equilibrium state away from vacuum. The local solutions for general initial data and global solutions for small initial data in various spaces are achieved in [14,46]. For arbitrary large initial data and the case that the height of fluid surface may vanish, the global weak entropy solution was obtained in [3,4,19,29]. Later on, for initial data allowing vacuum, the local existence of classical solution was obtained in [15], and the case of the degenerate viscosity was treated in [30].

The shallow water system is also regarded as an important extension of the two dimensional isentropic compressible Navier–Stokes equations with rotating force. There are numerous literatures on the existence and behavior of solutions to compressible Navier–Stokes equations with constant viscosity. For initial data close to a non-vacuum equilibrium, the existence of classical